

THE PROBLEMS OF PHILOSOPHY

VAGUENESS

TIMOTHY WILLIAMSON



**Also available as a printed book
see title verso for ISBN details**

Vagueness

If you keep removing single grains of sand from a heap, when is it no longer a heap? This question, and many others like it, soon lead us to the problem of vagueness.

Timothy Williamson traces the history of the problem from discussions of the heap paradox in ancient Greece to modern formal approaches, such as fuzzy logic. He discusses the view that classical logic and formal semantics do not apply to vague languages and shows that none of the alternative approaches can give a satisfying account of vagueness without falling back on classical logic.

Against this historical and critical background, Williamson then develops his own epistemicist position. Vagueness, he argues, is an epistemic phenomenon, a kind of ignorance: there really is a specific grain of sand whose removal turns the heap into a non-heap, but we cannot know which one it is.

Williamson's argument has ramifications far beyond the study of vagueness. It reasserts the validity of classical logic and semantics; more generally, it makes the thoroughly realist point that even the truth about the boundaries of our concepts can be beyond our capacity to know it.

The approach throughout keeps technicalities to a minimum; this is partly to counter the illusion, encouraged by the emphasis on formal systems, that vagueness can be studied in a precise metalanguage. For the technically minded, an appendix shows how the epistemic view can be formalised within the framework of epistemic logic.

Timothy Williamson is Professor of Logic and Metaphysics at the University of Edinburgh. He is the author of *Identity and Discrimination*.

The Problems of Philosophy

Founding editor: Ted Honderich

Editors: Tim Crane and Jonathan Wolff, *University College London*

This series addresses the central problems of philosophy. Each book gives a fresh account of a particular philosophical theme by offering two perspectives on the subject: the historical context and the author's own distinctive and original contribution.

The books are written to be accessible to students of philosophy and related disciplines, while taking the debate to a new level.

DEMOCRACY*

Ross Harrison

THE EXISTENCE OF THE
WORLD*

Reinhardt Grossman

NAMING AND REFERENCE

R. J. Nelson

EXPLAINING EXPLANATION*

David-Hillel Ruben

IF P, THEN Q*

David H. Sanford

SCEPTICISM

Christopher Hookway

HUMAN CONSCIOUSNESS

Alastair Hannay

THE IMPLICATIONS OF
DETERMINISM

Roy Weatherford

THE INFINITE*

A. W. Moore

KNOWLEDGE AND BELIEF

Frederic F. Schmitt

KNOWLEDGE OF THE
EXTERNAL WORLD

Bruce Aune

MORAL KNOWLEDGE*

Alan Goldman

MIND–BODY IDENTITY

THEORIES*

Cynthia Macdonald

THE NATURE OF ART*

A. L. Cothey

PERSONAL IDENTITY

Harold W. Noonan

POLITICAL FREEDOM

George G. Brenkert

THE RATIONAL

FOUNDATIONS OF ETHICS*

T. L. S. Sprigge

PRACTICAL REASONING*

Robert Audi

RATIONALITY*

Harold I. Brown

THOUGHT AND LANGUAGE*

J. M. Moravcsik

THE WEAKNESS OF THE WILL

Justine Gosling

THE MIND AND ITS WORLD*

Gregory McCulloch

PERCEPTION

Howard Robinson

THE NATURE OF GOD

Gerard Hughes

*Also available in paperback

Vagueness

Timothy Williamson



London and New York

First published 1994
by Routledge
11 New Fetter Lane, London EC4P 4E
29 West 35th Street, New York, NY 10001

This edition published in the Taylor & Francis e-Library, 2001.

Paperback edition 1996

© 1994 Timothy Williamson

All rights reserved. No part of this book may be reprinted or reproduced or utilized in any form or by any electronic, mechanical, or by any other means, now known or hereafter invented, including photocopying and recording, or in any information storage or retrieval system, without permission in writing from the publishers.

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data

A catalogue record for this book is available from the Library of Congress

ISBN 0-415-03331-4 (hbk)

ISBN 0-415-13980-5 (pbk)

ISBN 0-203-01426-X Master e-book ISBN

ISBN 0-203-17453-4 (Glassbook Format)

For Nathan Isaacs and Jeff Williamson

Contents

Preface	xi
Introduction	1
1 The early history of sorites paradoxes	8
1.1 <i>The first sorites</i>	8
1.2 <i>Chrysippian silence</i>	12
1.3 <i>Sorites arguments and Stoic logic</i>	22
1.4 <i>The sorites in later antiquity</i>	27
1.5 <i>The sorites after antiquity</i>	31
2 The ideal of precision	36
2.1 <i>The emergence of vagueness</i>	36
2.2 <i>Frege</i>	37
2.3 <i>Peirce</i>	46
2.4 <i>Russell</i>	52
3 The rehabilitation of vagueness	70
3.1 <i>Vagueness and ordinary language</i>	70
3.2 <i>The Black–Hempel debate</i>	73
3.3 <i>Family resemblances</i>	84
3.4 <i>Open texture</i>	89
4 Many-valued logic and degrees of truth	96
4.1 <i>Overview</i>	96
4.2 <i>Truth-functionality</i>	97
4.3 <i>Three-valued logic: beginnings</i>	102
4.4 <i>Three-valued logic: Halldén</i>	103
4.5 <i>Three-valued logic: Körner</i>	108
4.6 <i>Three-valued logic: second-order vagueness</i>	111
4.7 <i>Continuum-valued logic: a rationale</i>	113
4.8 <i>Continuum-valued logic: truth-tables</i>	114

4.9	<i>Fuzzy sets and fuzzy logic</i>	120
4.10	<i>Degree-theoretic treatments of sorites paradoxes</i>	123
4.11	<i>Comparatives and modifiers</i>	124
4.12	<i>Vague degrees of truth</i>	127
4.13	<i>Non-numerical degrees of truth</i>	131
4.14	<i>Degree-functionality</i>	135
4.15	<i>Appendix: axiomatizations of continuum-valued logic</i>	138
5	Supervaluations	142
5.1	<i>Incomplete meanings</i>	142
5.2	<i>Origins</i>	143
5.3	<i>Logic and semantics</i>	146
5.4	<i>The elusiveness of supertruth</i>	153
5.5	<i>Supervaluational degrees of truth</i>	154
5.6	<i>Supervaluations and higher-order vagueness</i>	156
5.7	<i>Truth and supertruth</i>	162
6	Nihilism	165
6.1	<i>Despair</i>	165
6.2	<i>Global nihilism</i>	166
6.3	<i>Local nihilism: appearances</i>	171
6.4	<i>Local nihilism: colours</i>	180
7	Vagueness as ignorance	185
7.1	<i>Bivalence and ignorance</i>	185
7.2	<i>Bivalence and truth</i>	187
7.3	<i>Omniscient speakers</i>	198
7.4	<i>The supervenience of vagueness</i>	201
7.5	<i>Meaning and use</i>	205
7.6	<i>Understanding</i>	209
7.7	<i>Decidable cases</i>	212
8	Inexact knowledge	216
8.1	<i>The explanatory task</i>	216
8.2	<i>The crowd</i>	217
8.3	<i>Margins for error</i>	226
8.4	<i>Conceptual sources of inexactness</i>	230
8.5	<i>Recognition of vague concepts</i>	234
8.6	<i>Indiscriminable differences</i>	237
8.7	<i>Inexact beliefs</i>	244

9	Vagueness in the world	248
	9.1 <i>Supervenience and vague facts</i>	248
	9.2 <i>Determinacy in the world</i>	249
	9.3 <i>Unclearly de re</i>	257
	Appendix The logic of clarity	270
	Notes	276
	References	307
	Index	320

Preface

This book originated in my attempts to refute its main thesis: that vagueness consists in our ignorance of the sharp boundaries of our concepts, and therefore requires no revision of standard logic. For years I took this epistemic view of vagueness to be obviously false, as most philosophers do. In 1988 Simon Blackburn, then editor of the journal *Mind*, asked me to review Roy Sorensen's intriguing book *Blindspots*, which includes a defence of the epistemic view. It did not persuade me; I could not see what makes us ignorant, and Sorensen offered no specific explanation. An alternative treatment of vagueness, supervaluationism, looked more or less adequate – unlike other popular alternatives, such as three-valued and fuzzy logic, which on technical grounds have always looked like blind alleys. However, I continued to think about the epistemic view, for the standard objections to it did not seem quite decisive. It was not clear that they did not assume a suspect connection between what is true and what we can verify. It then struck me that the notion of a margin for error could be used to give a specific explanation of ignorance of the sharp boundaries of our concepts, and the epistemic view began to look more plausible. A limited version of it was tentatively proposed in my book *Identity and Discrimination* (Oxford, Blackwell, 1990). The more closely the objections to it were analysed, the weaker they seemed. The next step was to focus on the fact that the meaning of vague expressions can be stated only in a language into which those expressions can be translated; it is a mistake to treat the language in which one theorizes about vagueness as though it were precise. Mark Sainsbury's inaugural lecture at King's College London, 'Concepts without Boundaries', helped to bring the significance

of this point home to me, although we used it in quite different ways. It permits the formulation of arguments against a wide range of non-epistemic views, including the supervenience that had previously looked adequate (my objection to it, however, is not the one made in Sainsbury's lecture). The balance of arguments seemed to have moved firmly onto the side of the epistemic view. A book-length treatment was clearly needed. This is the result.

Some of the research for this book was carried out in late 1990 whilst I was a Visiting Fellow at the Research School of Social Sciences of the Australian National University in Canberra. Many people helped to make the visit a success; Philip and Eileen Pettit stand out. Gratitude is also due to University College Oxford and Oxford University for allowing me extra leave of absence in that academic year.

Ted Honderich kindly permitted me to substitute a volume on vagueness in this series for one planned on another subject. One result of working on the past of the problem of vagueness for which I am particularly grateful is a better sense of the richness of Stoic logic. In this connection, I thank David Sedley for permission to quote translations from the first volume of a work he edited with A.A. Long, *The Hellenistic Philosophers* (Cambridge, Cambridge University Press, 1987).

Parts of Chapters 7 and 8 are drawn from two previously published articles of mine: 'Vagueness and ignorance', *Aristotelian Society*, suppl. 66 (1992), 145–62, and 'Inexact knowledge', *Mind*, 101 (1992), 217–42. I am grateful to the Aristotelian Society and The Mind Association for permission to use this material.

For written comments on predecessors of parts of this book, many thanks go to Michael Bacharach, Justin Broackes, Myles Burnyeat, Peter Carruthers, Bill Child, Jack Copeland, Dorothy Edgington, Timothy Endicott, Graeme Forbes, Brian Garrett, Bill Hart, Dominic Hyde, Frank Jackson, Rosanna Keefe, Peter Lipton, Andrei Marmor, Gregory McCulloch, Karina and Angus McIntosh, David Over, Peter Pagin, Philip Percival, Philip Pettit, Mark Sainsbury, David Sedley, Jonathan Sutton, Charles Travis and David Wiggins. Peter Simons replied to 'Vagueness and ignorance' in an enjoyable symposium at Reading, chaired by Mark Sainsbury. More people than I can name helped with critical questions after talks on the epistemic view of vagueness, inexact knowledge and related

topics at the universities of Bradford, Bristol, Cambridge (the Moral Sciences Club), Dundee, Edinburgh, Heidelberg, Leeds, Lisbon, London (University College), New England (Armidale), Nottingham, Oslo, Oxford, Queensland, Stirling, Stockholm and Uppsala, the Australian National University and Monash University, and to a meeting of the Lisbon Philosophical Society in May 1991, an Anglo–Polish Symposium on the Philosophy of Logic and Language at Oriel College Oxford in September 1991, the Second Workshop on Knowledge, Belief and Strategic Interaction at Castiglioncello in June 1992 and the Joint Session of the Aristotelian Society and the Mind Association at Reading University in July 1992. Invidiously, I pick out George Bealer and Peter Menzies, because there is a particularly direct causal link between their questions and sections of the book. Early versions of several chapters were used in classes at Oxford, and were considerably improved as a result; Ron Chrisley, Michael Martin and Roger Teichmann were particularly persistent questioners. I have also been helped by conversations with Maria Baghramian, João Branquinho, John Campbell, David Charles, Kit Fine, Olav Gjelsvik and Peter Strawson (not to mention anyone previously mentioned). Juliane Kerkhecker guided me through Lorenzo Valla’s Latin.

Those who know Elisabetta Perosino Williamson will guess how she helped in the writing of this book, and how much. It is dedicated to a great-uncle and an uncle, whose open-minded rationality (amongst other things) I rightly tried to imitate, with only mixed success.

Introduction

Logicians are often accused of treating language as though it were precise, and ignoring its vagueness. Their standards of valid and invalid reasoning are held to be good enough for artificial precise languages, but to break down when applied to the natural vague languages in which we actually reason about the world that we experience. A perfectly precise language for such reasoning is an idealization never to be realized. Although we can make our language less vague, we cannot make it perfectly precise. If we try to do so by stipulating what our words are to mean, our stipulations will themselves be made in less than perfectly precise terms, and the reformed language will inherit some of that vagueness.

The problem is not confined to logic. Attempts to describe the semantics of natural languages in formal terms are also frequently supposed to ignore vagueness, and therefore to misdescribe the meanings of ordinary expressions. Of course, a theory might ignore vagueness and remain a useful approximation for some purposes, but it is also legitimate to ask what changes of theory are needed to take vagueness into account.

At the core of classical (i.e. standard) logic and semantics is the principle of bivalence, according to which every statement is either true or false. This is the principle most obviously threatened by vagueness. When, for example, did Rembrandt become old? For each second of his life, one can consider the statement that he was old then. Some of those statements are false; others are true. If all of them are true or false, then there was a last second at which it was false to say that Rembrandt was old, immediately followed by a first second at which it was true to say that he was old. Which second was that? We have no way of knowing. Indeed, it is widely felt to be

just silly to suppose that there was such a second. Our use of the word 'old' is conceived as too vague to single one out. On such grounds, the principle of bivalence has been rejected for vague languages. To reject bivalence is to reject classical logic or semantics.

At some times, it was unclear whether Rembrandt was old. He was neither clearly old nor clearly not old. The unclarity resulted from vagueness in the statement that Rembrandt was old. We can even use such examples to define the notion of vagueness. An expression or concept is vague if and only if it can result in unclarity of the kind just exemplified. Such a definition does not pretend to display the underlying nature of the phenomenon. In particular, it does not specify whether the unclarity results from the failure of the statement to be true or false, or simply from our inability to find out which. The definition is neutral on such points of theory. Just as we might agree to define the term 'light', or 'poetry', by examples, in order not to talk past each other when disagreeing about the nature of light, or poetry, so we can agree to define the term 'vagueness' by examples, in order not to talk past each other when disagreeing about the nature of vagueness.

The phenomenon of vagueness is broad. Most challenges to classical logic or semantics depend on special features of a subject matter: the future, the infinite, the quantum mechanical. For all such a challenge implies, classical logic and semantics apply to statements about other subject matters. Vagueness, in contrast, presents a ubiquitous challenge. It is hard to make a perfectly precise statement about anything. If classical logic and semantics apply only to perfectly precise languages, then they apply to no language that we can speak.

The phenomenon of vagueness is deep as well as broad. It would be shallow if it could be adequately described in precise terms. That is not generally possible. The difficulties presented by the question 'When did Rembrandt become old?' are also presented by the question 'When did Rembrandt become clearly old?'. At some times, it was unclear whether it was unclear whether Rembrandt was old. The limits of vagueness are themselves vague. The same difficulties are presented by the question 'When did Rembrandt become clearly clearly old?'; the point reiterates *ad infinitum*. This is the phenomenon of higher-order vagueness. It means that

the meta-language in which we describe the vagueness of a vague language will itself be vague.

The use of non-classical systems of logic or semantics has been advocated for vague languages. New and increasingly complex systems continue to be invented. What none has so far given is a satisfying account of higher-order vagueness. In more or less subtle ways, the meta-language is treated as though it were precise. For example, classical logic is said to be invalid for vague languages, and is then used in the meta-language. Such proposals underestimate the depth of the problem.

The problem is not solved by the pessimistic idea that no system of logic or semantics, classical or non-classical, is adequate for a vague language. That idea still permits one to ask for perspicuous descriptions of vagueness in particular cases. No one has given a satisfying and perspicuous description of higher-order vagueness without use of classical logic. Of course, the nature of vagueness might be to defy perspicuous description, but that counsel of despair should prevail only if there is good evidence that it does not overestimate the depth of the problem.

The thesis of this book is that vagueness is an epistemic phenomenon. As such, it constitutes no objection to classical logic or semantics. In cases of unclarity, statements remain true or false, but speakers of the language have no way of knowing which. Higher-order vagueness consists in ignorance about ignorance.

At first sight, the epistemic view of vagueness is incredible. We may think that we cannot conceive how a vague statement could be true or false in an unclear case. For when we conceive that something is so, we tend to imagine finding out that it is so. We are uneasy with a fact on which we cannot attain such a first-personal perspective. We have no idea how we ever could have found out that the vague statement is true, or that it is false, in an unclear case; we are consequently unable to imagine finding out that it is true, or that it is false; we fallaciously conclude that it is inconceivable that it is true, and inconceivable that it is false. Such fallacies of the imagination must be laid aside before the epistemic view can be adequately assessed.

Most work on the problem of vagueness assumes that the epistemic view is false, without seriously arguing the point. If the epistemic view is true, that work is fundamentally mistaken. Even if the epistemic view is

false, that work is ungrounded until cogent arguments against the view have been found. The assessment of the epistemic view is therefore one of the main tasks facing the study of vagueness. This book contributes to that task.

The assessment of the epistemic view has ramifications far beyond the study of vagueness. As already noted, classical logic and semantics are at stake. More generally, the epistemic view implies a form of realism, that even the truth about the boundaries of our concepts can outrun our capacity to know it. To deny the epistemic view of vagueness is therefore to impose limits on realism; to assert it is to endorse realism in a thoroughgoing form.

The first part of the book is historical. It traces the slow and intermittent recognition of vagueness as a distinct and problematic phenomenon, up to the origins of the theories of vagueness that have been popular over the last two decades. This part is also critical. It argues that none of the extant non-epistemic theories of vagueness is adequate. Not only do they abandon classical logic or semantics for alternatives of doubtful coherence; those sacrifices are not rewarded by adequate insight into the nature of vagueness. The second part of the book is constructive. It develops and applies an epistemic view of vagueness, finds the standard objections to it fallacious, and concludes that the epistemic view provides the best explanation of the phenomenon of vagueness.

The Greeks introduced the problem of vagueness into philosophy, in the guise of the original sorites paradox: if the removal of one grain from a heap always leaves a heap, then the successive removal of every grain still leaves a heap. Chapter 1 sketches the history of this paradox and its variants from their invention to the nineteenth century. Stoic logicians are interpreted as taking an epistemic view of sorites paradoxes.

What makes a sorites paradox paradoxical is the vagueness of its central term, e.g. 'heap'. Historically, however, such paradoxes were identified by their form. Vagueness as such became a topic of philosophical discussion only at the start of the twentieth century, when it presented an obstacle to the ideal of a logically perfect language associated with the development of modern logic. Only with difficulty was the phenomenon of unclear boundaries separated from other phenomena, such as lack of specificity, to which the term 'vagueness' is

also applied in everyday usage. Chapter 2 discusses three stages in the emergence of the philosophical concept of vagueness, in the work of Frege, Peirce and Russell.

As philosophical attention turned to ordinary language, vagueness acquired a more positive image. It was seen no longer as a deviation from an ideal norm, but as the real norm itself. As such, it was described by Black, Wittgenstein and Waismann. Their work is discussed in Chapter 3.

Formal treatments of vagueness have become common only in the last few decades. One main approach relies on many-valued logic, which replaces the dichotomy of truth and falsity by a manyfold classification. Chapter 4 follows the development of its application to the problem of vagueness, from the use of three-valued logic to the growth of 'fuzzy logic' and other logics based on infinitely many values, and then of more sophisticated accounts appealing to a qualitative conception of degrees of truth. These views are criticized on several grounds. None has adequately treated higher-order vagueness; degrees of truth are not connected with vagueness in the requisite way; the generalization from two-valued logic to many-valued logic has highly counter-intuitive consequences when applied to natural languages.

A technically subtler approach to vagueness is supervaluationism, with which Chapter 5 is concerned. It preserves almost all of classical logic, at the expense of classical semantics, by giving a non-standard account of truth. It also treats higher-order vagueness in a promising way. However, it is argued that the treatment of higher-order vagueness undermines the non-standard account of truth, making supervaluationism as a whole unmotivated.

On a more pessimistic view, vagueness is a form of incoherence. If this view is taken globally, Chapter 6 suggests, all rational discourse is subverted, for vagueness is ubiquitous. However, local forms of nihilism might be coherent. They have been defended in the special case of concepts used to describe perceptual appearances, on the grounds that such concepts cannot differentiate between perceptually indiscriminable items, yet perceptually discriminable items can be linked by a sorites series of which each member is perceptually indiscriminable from its neighbours. However, careful attention to the structure of the relevant concepts shows

that the paradoxical arguments are unsound. In particular, they falsely assume that appearances are just what they appear to be.

Chapter 7 defends the epistemic view of vagueness. First, it argues that it is incoherent to deny the principle of bivalence for vague statements in unclear cases. It then questions our ability to think through coherently the consequences of a non-epistemic view of vagueness. Obvious objections to the epistemic view are analysed and shown to be fallacious. A picture of linguistic understanding is sketched, on which we can know that a word has a given meaning without knowing what the boundaries of that meaning are in conceptual space.

Chapter 8 develops the epistemological background to the epistemic view. It gives independent justification for principles about knowledge on which the ignorance postulated by the view was only to be expected as a special case of a much wider phenomenon, inexact knowledge. Nevertheless, the case is special, for the source of the inexactness is distinctive in being conceptual. Higher-order vagueness has a central place in this account, for a central feature of inexact knowledge is that one can know something without being in a position to know that one knows it; when the inexactness takes the form of vagueness, this becomes unclarity about unclarity. The epistemology of inexact knowledge is then used to analyse in greater depth the phenomena of indiscriminability to which the nihilist appeals.

It is controversial whether in any sense the world itself, as opposed to our representations of it, can be vague. Chapter 9 examines the issue. It argues that the epistemic view permits objects to be vague in a modest sense, for the impossibility of knowing their boundaries may be independent of the way in which the objects are represented.

The Appendix identifies the formal system uniquely appropriate for the logic of clarity and unclarity on the epistemic view of vagueness.

Most of the book keeps technicalities to a minimum. The gain in intelligibility will, it is hoped, outweigh the loss in rigour. There is also a philosophical reason for minimizing technicality. The emphasis on formal systems has encouraged the illusion that vagueness can be studied in a precise meta-language. It has therefore caused the significance of higher-order vagueness to be underestimated. Indeed, to use a supposedly precise meta-language in studying vague terms is to use a language into which, by

hypothesis, they cannot be translated. Since vague terms are meaningful, this is an expressive limitation on the meta-language. It is not an innocent one. The argument in Chapter 7 for the incoherence of denials of bivalence in unclear cases can be stated only in a language into which the relevant vague terms can be translated. To deny bivalence is in the end to treat vague utterances as though they said nothing. Vagueness can be understood only from within.

The early history of sorites paradoxes

1.1 THE FIRST SORITES¹

The logician Eubulides of Miletus, a contemporary of Aristotle, was famous for seven puzzles. One was the Liar: if a man says that he is lying, is he telling the truth? Another was the Hooded Man: how can you know your brother when you do not know that hooded man, who is in fact your brother? The Electra turned on the delusion of Orestes in his madness, who took his sister Electra for a Fury. The Elusive Man will appear later. There was also the Horned Man: since you still have what you have not lost, and you have not lost horns, you still have them (hence the horns of a dilemma). The remaining puzzles were the Bald Man and (accompanying five men and one woman) the Heap. In antiquity they were usually formulated as series of questions.²

Does one grain of wheat make a heap? Do two grains of wheat make a heap? Do three grains of wheat make a heap? . . . Do ten thousand grains of wheat make a heap? It is to be understood that the grains are properly piled up, and that a heap must contain reasonably many grains. If you admit that one grain does not make a heap, and are unwilling to make a fuss about the addition of any single grain, you are eventually forced to admit that ten thousand grains do not make a heap.

Is a man with one hair on his head bald? Is a man with two hairs on his head bald? Is a man with three hairs on his head bald? . . . Is a man with ten thousand hairs on his head bald? It is to be understood that the hairs are properly distributed, and that a man with reasonably few hairs is bald. If you admit that a man with one hair is bald, and are unwilling to make a fuss about the addition of any single hair, you are eventually forced to admit that a man with ten thousand hairs is bald.

The standard ancient terms for the Heap and the Bald Man were ‘*sorites*’ and ‘*phalakros*’ respectively. The Greek adjective ‘*sorites*’ comes from the noun ‘*soros*’, for ‘heap’, and means literally *heaper*, as in ‘the heaper paradox’. Perhaps Eubulides coined the word himself. The primary reference was to the content of the puzzle, although there may also have been a secondary allusion to the heaping up of questions in its form. The term was extended to similar puzzles, such as the Bald Man. They were also known as little-by-little arguments.³

So far as is known, Eubulides invented the Heap and the Bald Man himself. The Heap may have been inspired by a different puzzle, the Millet Seed, propounded by Zeno of Elea a century earlier. If the fall of a seed to the ground were completely silent, so would be the fall of a bushel of seed, which it is not; thus each seed must make a noise when it falls to the ground. That puzzle can certainly be adapted to sorites form. Does one seed make a noise when it falls to the ground? Do two seeds? Three? . . . However, Zeno seems to have based his puzzle on something much more specific: a principle that the noise made when some grains fall to the ground is proportional to their weight.⁴ Eubulides may well have been the first to focus on the general difficulty that sorites questioning presents; he is unlikely to have overlooked the common form of the Heap and the Bald Man, however much their difference in content caused them to be listed as distinct puzzles.

It is not known what Eubulides used sorites puzzles for – fun, troublemaking or some graver purpose. Many philosophical doctrines have been suggested as the target he intended them to destroy: the coherence of empirical concepts (such as ‘bald’ and ‘heap’), the law of non-contradiction (‘Not both P and not P ’), the law of excluded middle (‘Either P or not P ’), pluralism (the existence of more than one thing), Aristotle’s theory of infinity (as potential rather than actual), Aristotle’s theory of the mean (as the place of virtue between vicious extremes). The evidence gives little support to any of these suggestions. Eubulides is indeed said to have attacked Aristotle, but in slanderous terms; the sources do not connect the dispute with any of the puzzles. Aristotle betrays no clear awareness of sorites reasoning in any of his extant works. Some later commentators did consider its use against Aristotle’s theory of the mean, but without suggesting that either Eubulides or Aristotle had done so. Eubulides’

