

Chapter 1

- 1.1 $(2/3)^{10} = 0.0173$ yd; $6(2/3)^{10} = 0.104$ yd (compared to a total of 5 yd)
 1.3 $5/9$ 1.4 $9/11$ 1.5 $7/12$
 1.6 $11/18$ 1.7 $5/27$ 1.8 $25/36$
 1.9 $6/7$ 1.10 $15/26$ 1.11 $19/28$
 1.13 \$1646.99 1.15 Blank area = 1
 1.16 At $x = 1$: $1/(1+r)$; at $x = 0$: $r/(1+r)$; maximum escape at $x = 0$ is $1/2$.

- 2.1 1 2.2 $1/2$ 2.3 0
 2.4 ∞ 2.5 0 2.6 ∞
 2.7 e^2 2.8 0 2.9 1

- 4.1 $a_n = 1/2^n \rightarrow 0$; $S_n = 1 - 1/2^n \rightarrow 1$; $R_n = 1/2^n \rightarrow 0$
 4.2 $a_n = 1/5^{n-1} \rightarrow 0$; $S_n = (5/4)(1 - 1/5^n) \rightarrow 5/4$; $R_n = 1/(4 \cdot 5^{n-1}) \rightarrow 0$
 4.3 $a_n = (-1/2)^{n-1} \rightarrow 0$; $S_n = (2/3)[1 - (-1/2)^n] \rightarrow 2/3$; $R_n = (2/3)(-1/2)^n \rightarrow 0$
 4.4 $a_n = 1/3^n \rightarrow 0$; $S_n = (1/2)(1 - 1/3^n) \rightarrow 1/2$; $R_n = 1/(2 \cdot 3^n) \rightarrow 0$
 4.5 $a_n = (3/4)^{n-1} \rightarrow 0$; $S_n = 4[1 - (3/4)^n] \rightarrow 4$; $R_n = 4(3/4)^n \rightarrow 0$
 4.6 $a_n = \frac{1}{n(n+1)} \rightarrow 0$; $S_n = 1 - \frac{1}{n+1} \rightarrow 1$; $R_n = \frac{1}{n+1} \rightarrow 0$
 4.7 $a_n = (-1)^{n+1} \left(\frac{1}{n} + \frac{1}{n+1} \right) \rightarrow 0$; $S_n = 1 + \frac{(-1)^{n+1}}{n+1} \rightarrow 1$; $R_n = \frac{(-1)^n}{n+1} \rightarrow 0$

- 5.1 D 5.2 Test further 5.3 Test further
 5.4 D 5.5 D 5.6 Test further
 5.7 Test further 5.8 Test further
 5.9 D 5.10 D

- 6.5 (a) D 6.5 (b) D

Note: In the following answers, $I = \int_{-\infty}^{\infty} a_n dn$; $\rho =$ test ratio.

- 6.7 D, $I = \infty$ 6.8 D, $I = \infty$ 6.9 C, $I = 0$
 6.10 C, $I = \pi/6$ 6.11 C, $I = 0$ 6.12 C, $I = 0$
 6.13 D, $I = \infty$ 6.14 D, $I = \infty$ 6.18 D, $\rho = 2$
 6.19 C, $\rho = 3/4$ 6.20 C, $\rho = 0$ 6.21 D, $\rho = 5/4$
 6.22 C, $\rho = 0$ 6.23 D, $\rho = \infty$ 6.24 D, $\rho = 9/8$
 6.25 C, $\rho = 0$ 6.26 C, $\rho = (e/3)^3$ 6.27 D, $\rho = 100$
 6.28 C, $\rho = 4/27$ 6.29 D, $\rho = 2$ 6.31 D, cf. $\sum n^{-1}$
 6.32 D, cf. $\sum n^{-1}$ 6.33 C, cf. $\sum 2^{-n}$ 6.34 C, cf. $\sum n^{-2}$
 6.35 C, cf. $\sum n^{-2}$ 6.36 D, cf. $\sum n^{-1/2}$

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|-----|---|-----|---|-----|---|-----|---|
| 7.1 | C | 7.2 | D | 7.3 | C | 7.4 | C |
| 7.5 | C | 7.6 | D | 7.7 | C | 7.8 | C |
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|------|--|-------|--|
| 9.1 | D, cf. $\sum n^{-1}$ | 9.2 | D, $a_n \neq 0$ |
| 9.3 | C, $I = 0$ | 9.4 | D, $I = \infty$, or cf. $\sum n^{-1}$ |
| 9.5 | C, cf. $\sum n^{-2}$ | 9.6 | C, $\rho = 1/4$ |
| 9.7 | D, $\rho = 4/3$ | 9.8 | C, $\rho = 1/5$ |
| 9.9 | D, $\rho = e$ | 9.10 | D, $a_n \neq 0$ |
| 9.11 | D, $I = \infty$, or cf. $\sum n^{-1}$ | 9.12 | C, cf. $\sum n^{-2}$ |
| 9.13 | C, $I = 0$, or cf. $\sum n^{-2}$ | 9.14 | C, alt. ser. |
| 9.15 | D, $\rho = \infty$, $a_n \neq 0$ | 9.16 | C, cf. $\sum n^{-2}$ |
| 9.17 | C, $\rho = 1/27$ | 9.18 | C, alt. ser. |
| 9.19 | C | 9.20 | C |
| 9.21 | C, $\rho = 1/2$ | | |
| 9.22 | (a) C | (b) D | (c) $k > e$ |
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|-------|-----------------------------------|-------|---------------------|-------|-------------------------|
| 10.1 | $ x < 1$ | 10.2 | $ x < 3/2$ | 10.3 | $ x \leq 1$ |
| 10.4 | $ x \leq \sqrt{2}$ | 10.5 | All x | 10.6 | All x |
| 10.7 | $-1 \leq x < 1$ | 10.8 | $-1 < x \leq 1$ | 10.9 | $ x < 1$ |
| 10.10 | $ x \leq 1$ | 10.11 | $-5 \leq x < 5$ | 10.12 | $ x < 1/2$ |
| 10.13 | $-1 < x \leq 1$ | 10.14 | $ x < 3$ | 10.15 | $-1 < x < 5$ |
| 10.16 | $-1 < x < 3$ | 10.17 | $-2 < x \leq 0$ | 10.18 | $-3/4 \leq x \leq -1/4$ |
| 10.19 | $ x < 3$ | 10.20 | All x | 10.21 | $0 \leq x \leq 1$ |
| 10.22 | No x | 10.23 | $x > 2$ or $x < -4$ | 10.24 | $ x < \sqrt{5}/2$ |
| 10.25 | $n\pi - \pi/6 < x < n\pi + \pi/6$ | | | | |

$$13.4 \quad \binom{-1/2}{0} = 1; \quad \binom{-1/2}{n} = \frac{(-1)^n(2n-1)!!}{(2n)!!}$$

Answers to part (b), Problems 5 to 19:

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|-------|--|-------|---|
| 13.5 | $-\sum_1^{\infty} \frac{x^{n+2}}{n}$ | 13.6 | $\sum_0^{\infty} \binom{1/2}{n} x^{n+1}$ (see Example 2) |
| 13.7 | $\sum_0^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$ | 13.8 | $\sum_0^{\infty} \binom{-1/2}{n} (-x^2)^n$ (see Problem 13.4) |
| 13.9 | $1 + 2 \sum_1^{\infty} x^n$ | 13.10 | $\sum_0^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$ |
| 13.11 | $\sum_0^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$ | 13.12 | $\sum_0^{\infty} \frac{(-1)^n x^{4n+1}}{(2n)!(4n+1)}$ |
| 13.13 | $\sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}$ | 13.14 | $\sum_0^{\infty} \frac{x^{2n+1}}{2n+1}$ |
| 13.15 | $\sum_0^{\infty} \binom{-1/2}{n} (-1)^n \frac{x^{2n+1}}{2n+1}$ | | |
| 13.16 | $\sum_0^{\infty} \frac{x^{2n}}{(2n)!}$ | 13.17 | $2 \sum_{\text{odd } n}^{\infty} \frac{x^n}{n}$ |
| 13.18 | $\sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$ | 13.19 | $\sum_0^{\infty} \binom{-1/2}{n} \frac{x^{2n+1}}{2n+1}$ |
| 13.20 | $x + x^2 + x^3/3 - x^5/30 - x^6/90 \dots$ | | |
| 13.21 | $x^2 + 2x^4/3 + 17x^6/45 \dots$ | | |
| 13.22 | $1 + 2x + 5x^2/2 + 8x^3/3 + 65x^4/24 \dots$ | | |
| 13.23 | $1 - x + x^3 - x^4 + x^6 \dots$ | | |

- 13.24 $1 + x^2/2! + 5x^4/4! + 61x^6/6! \dots$
 13.25 $1 - x + x^2/3 - x^4/45 \dots$
 13.26 $1 + x^2/4 + 7x^4/96 + 139x^6/5760 \dots$
 13.27 $1 + x + x^2/2 - x^4/8 - x^5/15 \dots$
 13.28 $x - x^2/2 + x^3/6 - x^5/12 \dots$
 13.29 $1 + x/2 - 3x^2/8 + 17x^3/48 \dots$
 13.30 $1 - x + x^2/2 - x^3/2 + 3x^4/8 - 3x^5/8 \dots$
 13.31 $1 - x^2/2 - x^3/2 - x^4/4 - x^5/24 \dots$
 13.32 $x + x^2/2 - x^3/6 - x^4/12 \dots$
 13.33 $1 + x^3/6 + x^4/6 + 19x^5/120 + 19x^6/120 \dots$
 13.34 $x - x^2 + x^3 - 13x^4/12 + 5x^5/4 \dots$
 13.35 $1 + x^2/3! + 7x^4/(3 \cdot 5!) + 31x^6/(3 \cdot 7!) \dots$
 13.36 $u^2/2 + u^4/12 + u^6/20 \dots$
 13.37 $-(x^2/2 + x^4/12 + x^6/45 \dots)$
 13.38 $e(1 - x^2/2 + x^4/6 \dots)$
 13.39 $1 - (x - \pi/2)^2/2! + (x - \pi/2)^4/4! \dots$
 13.40 $1 - (x - 1) + (x - 1)^2 - (x - 1)^3 \dots$
 13.41 $e^3[1 + (x - 3) + (x - 3)^2/2! + (x - 3)^3/3! \dots]$
 13.42 $-1 + (x - \pi)^2/2! - (x - \pi)^4/4! \dots$
 13.43 $-[(x - \pi/2) + (x - \pi/2)^3/3 + 2(x - \pi/2)^5/15 \dots]$
 13.44 $5 + (x - 25)/10 - (x - 25)^2/10^3 + (x - 25)^3/(5 \cdot 10^4) \dots$
- 14.6 Error $< (1/2)(0.1)^2 \div (1 - 0.1) < 0.0056$
 14.7 Error $< (3/8)(1/4)^2 \div (1 - \frac{1}{4}) = 1/32$
 14.8 For $x < 0$, error $< (1/64)(1/2)^4 < 0.001$
 For $x > 0$, error $< 0.001 \div (1 - \frac{1}{2}) = 0.002$
 14.9 Term $n + 1$ is $a_{n+1} = \frac{1}{(n+1)(n+2)}$, so $R_n = (n + 2)a_{n+1}$.
 14.10 $S_4 = 0.3052$, error < 0.0021 (cf. $S = 1 - \ln 2 = 0.307$)
- 15.1 $-x^4/24 - x^5/30 \dots \simeq -3.376 \times 10^{-16}$
 15.2 $x^8/3 - 14x^{12}/45 \dots \simeq 1.433 \times 10^{-16}$
 15.3 $x^5/15 - 2x^7/45 \dots \simeq 6.667 \times 10^{-17}$
 15.4 $x^3/3 + 5x^4/6 \dots \simeq 1.430 \times 10^{-11}$
- 15.5 0 15.6 12 15.7 10!
 15.8 1/2 15.9 -1/6 15.10 -1
 15.11 4 15.12 1/3 15.13 -1
 15.14 $t - t^3/3$, error $< 10^{-6}$ 15.15 $\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}$, error $< \frac{1}{7}10^{-7}$
 15.16 $e^2 - 1$ 15.17 $\cos \frac{\pi}{2} = 0$
 15.18 $\ln 2$ 15.19 $\sqrt{2}$
- 15.20 (a) 1/8 (b) $5e$ (c) 9/4
 15.21 (a) 0.397117 (b) 0.937548 (c) 1.291286
 15.22 (a) $\pi^4/90$ (b) 1.202057 (c) 2.612375
 15.23 (a) 1/2 (b) 1/6 (c) 1/3 (d) -1/2
 15.24 (a) $-\pi$ (b) 0 (c) -1
 (d) 0 (e) 0 (f) 0
- 15.27 (a) $1 - \frac{v}{c} = 1.3 \times 10^{-5}$, or $v = 0.999987c$
 (b) $1 - \frac{v}{c} = 5.2 \times 10^{-7}$
 (c) $1 - \frac{v}{c} = 2.1 \times 10^{-10}$
 (d) $1 - \frac{v}{c} = 1.3 \times 10^{-11}$
- 15.28 $mc^2 + \frac{1}{2}mv^2$
 15.29 (a) $F/W = \theta + \theta^3/3 \dots$
 (b) $F/W = x/l + x^3/(2l^3) + 3x^5/(8l^5) \dots$

- 15.30 (a) $T = F(5/x + x/40 - x^3/16000 \dots)$
 (b) $T = \frac{1}{2}(F/\theta)(1 + \theta^2/6 + 7\theta^4/360 \dots)$
- 15.31 (a) finite (b) infinite
- 16.1 (c) overhang: 2 3 10 100
 books needed: 32 228 2.7×10^8 4×10^{86}
- 16.4 C, $\rho = 0$ 16.5 D, $a_n \not\rightarrow 0$ 16.6 C, cf. $\sum n^{-3/2}$
 16.7 D, $I = \infty$ 16.8 D, cf. $\sum n^{-1}$ 16.9 $-1 \leq x < 1$
 16.10 $|x| < 4$ 16.11 $|x| \leq 1$ 16.12 $|x| < 5$
 16.13 $-5 < x \leq 1$
 16.14 $1 - x^2/2 + x^3/2 - 5x^4/12 \dots$
 16.15 $-x^2/6 - x^4/180 - x^6/2835 \dots$
 16.16 $1 - x/2 + 3x^2/8 - 11x^3/48 + 19x^4/128 \dots$
 16.17 $1 + x^2/2 + x^4/4 + 7x^6/48 \dots$
 16.18 $x - x^3/3 + x^5/5 - x^7/7 \dots$
 16.19 $-(x - \pi) + (x - \pi)^3/3! - (x - \pi)^5/5! \dots$
 16.20 $2 + (x - 8)/12 - (x - 8)^2/(2^5 \cdot 3^2) + 5(x - 8)^3/(2^8 \cdot 3^4) \dots$
 16.21 $e[1 + (x - 1) + (x - 1)^2/2! + (x - 1)^3/3! \dots]$
 16.22 $\arctan 1 = \pi/4$ 16.23 $1 - (\sin \pi)/\pi = 1$
 16.24 $e^{\ln 3} - 1 = 2$ 16.25 -2
 16.26 $-1/3$ 16.27 $2/3$
 16.28 1 16.29 $6!$
- 16.30 (b) For $N = 130$, $10.5821 < \zeta(1.1) < 10.5868$
 16.31 (a) 10^{430} terms. For $N = 200$, $100.5755 < \zeta(1.01) < 100.5803$
 16.31 (b) 2.66×10^{86} terms. For $N = 15$, $1.6905 < S < 1.6952$
 16.31 (c) $e^{200} = 10^{3.1382 \times 10^{86}}$ terms. For $N = 40$, $38.4048 < S < 38.4088$

Chapter 2

	x	y	r	θ
4.1	1	1	$\sqrt{2}$	$\pi/4$
4.2	-1	1	$\sqrt{2}$	$3\pi/4$
4.3	1	$-\sqrt{3}$	2	$-\pi/3$
4.4	$-\sqrt{3}$	1	2	$5\pi/6$
4.5	0	2	2	$\pi/2$
4.6	0	-4	4	$-\pi/2$
4.7	-1	0	1	π
4.8	3	0	3	0
4.9	-2	2	$2\sqrt{2}$	$3\pi/4$
4.10	2	-2	$2\sqrt{2}$	$-\pi/4$
4.11	$\sqrt{3}$	1	2	$\pi/6$
4.12	-2	$-2\sqrt{3}$	4	$-2\pi/3$
4.13	0	-1	1	$3\pi/2$
4.14	$\sqrt{2}$	$\sqrt{2}$	2	$\pi/4$
4.15	-1	0	1	$-\pi$ or π
4.16	5	0	5	0
4.17	1	-1	$\sqrt{2}$	$-\pi/4$
4.18	0	3	3	$\pi/2$
4.19	4.69	1.71	5	$20^\circ = 0.35$
4.20	-2.39	-6.58	7	$-110^\circ = -1.92$
5.1	1/2	-1/2	$1/\sqrt{2}$	$-\pi/4$
5.2	-1/2	-1/2	$1/\sqrt{2}$	$-3\pi/4$ or $5\pi/4$
5.3	1	0	1	0
5.4	0	2	2	$\pi/2$
5.5	2	$2\sqrt{3}$	4	$\pi/3$
5.6	-1	0	1	π
5.7	7/5	-1/5	$\sqrt{2}$	$-8.13^\circ = -0.14$
5.8	1.6	-2.7	3.14	$-59.3^\circ = -1.04$
5.9	-10.4	22.7	25	$2 = 114.6^\circ$
5.10	-25/17	19/17	$\sqrt{58/17}$	$142.8^\circ = 2.49$
5.11	17	-12	20.8	$-35.2^\circ = -0.615$
5.12	2.65	1.41	3	$28^\circ = 0.49$
5.13	1.55	4.76	5	$2\pi/5$
5.14	1.27	-2.5	2.8	$-1.1 = -63^\circ$
5.15	21/29	-20/29	1	$-43.6^\circ = -0.76$
5.16	1.53	-1.29	2	$-40^\circ = -0.698$
5.17	-7.35	-10.9	13.1	$-124^\circ = -2.16$
5.18	-0.94	-0.36	1	201° or -159° , 3.51 or -2.77

- 9.1 $(1-i)/\sqrt{2}$ 9.2 i 9.3 $-9i$
 9.4 $-e(1+i\sqrt{3})/2$ 9.5 -1 9.6 1
 9.7 $3e^2$ 9.8 $-\sqrt{3}+i$ 9.9 $-2i$
 9.10 -2 9.11 $-1-i$ 9.12 $-2-2i\sqrt{3}$
 9.13 $-4+4i$ 9.14 64 9.15 $2i-4$
 9.16 $-2\sqrt{3}-2i$ 9.17 $-(1+i)/4$ 9.18 1
 9.19 16 9.20 i 9.21 1
 9.22 $-i$ 9.23 $(\sqrt{3}+i)/4$ 9.24 $4i$
 9.25 -1 9.26 $(1+i\sqrt{3})/2$ 9.29 1
 9.30 $e^{\sqrt{3}}$ 9.31 5 9.32 $3e^2$
 9.33 $2e^3$ 9.34 $4/e$ 9.35 21
 9.36 4 9.37 1 9.38 $1/\sqrt{2}$
- 10.1 $1, (-1 \pm i\sqrt{3})/2$ 10.2 $3, 3(-1 \pm i\sqrt{3})/2$
 10.3 $\pm 1, \pm i$ 10.4 $\pm 2, \pm 2i$
 10.5 $\pm 1, (\pm 1 \pm i\sqrt{3})/2$ 10.6 $\pm 2, \pm 1 \pm i\sqrt{3}$
 10.7 $\pm\sqrt{2}, \pm i\sqrt{2}, \pm 1 \pm i$ 10.8 $\pm 1, \pm i, (\pm 1 \pm i)/\sqrt{2}$
 10.9 $1, 0.309 \pm 0.951i, -0.809 \pm 0.588i$
 10.10 $2, 0.618 \pm 1.902i, -1.618 \pm 1.176i$
 10.11 $-2, 1 \pm i\sqrt{3}$ 10.12 $-1, (1 \pm i\sqrt{3})/2$
 10.13 $\pm 1 \pm i$ 10.14 $(\pm 1 \pm i)/\sqrt{2}$
 10.15 $\pm 2i, \pm\sqrt{3} \pm i$ 10.16 $\pm i, (\pm\sqrt{3} \pm i)/2$
 10.17 $-1, 0.809 \pm 0.588i, -0.309 \pm 0.951i$
 10.18 $\pm(1+i)/\sqrt{2}$ 10.19 $-i, (\pm\sqrt{3}+i)/2$
 10.20 $2i, \pm\sqrt{3}-i$ 10.21 $\pm(\sqrt{3}+i)$
 10.22 $r = \sqrt{2}, \theta = 45^\circ + 120^\circ n: 1+i, -1.366+0.366i, 0.366-1.366i$
 10.23 $r = 2, \theta = 30^\circ + 90^\circ n: \pm(\sqrt{3}+i), \pm(1-i\sqrt{3})$
 10.24 $r = 1, \theta = 30^\circ + 45^\circ n:$
 $\pm(\sqrt{3}+i)/2, \pm(1-i\sqrt{3})/2, \pm(0.259+0.966i), \pm(0.966-0.259i)$
 10.25 $r = \sqrt[10]{2}, \theta = 45^\circ + 72^\circ n: 0.758(1+i), -0.487+0.955i,$
 $-1.059-0.168i, -0.168-1.059i, 0.955-0.487i$
 10.26 $r = 1, \theta = 18^\circ + 72^\circ n: i, \pm 0.951+0.309i, \pm 0.588-0.809i$
 10.28 $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$
 $\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$
- 11.3 $3(1-i)/\sqrt{2}$ 11.4 -8 11.5 $1+i$ 11.6 $13/5$
 11.7 $3i/5$ 11.8 $-41/9$ 11.9 $4i/3$ 11.10 -1
- 12.20 $\cosh 3z = \cosh^3 z + 3 \cosh z \sinh^2 z, \sinh 3z = 3 \cosh^2 z \sinh z + \sinh^3 z$
 12.22 $\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, \cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$
 12.23 $\cos x, |\cos x|$
 12.24 $\cosh x$
 12.25 $\sin x \cosh y - i \cos x \sinh y, \sqrt{\sin^2 x + \sinh^2 y}$
 12.26 $\cosh 2 \cos 3 - i \sinh 2 \sin 3 = -3.725 - 0.512i, 3.760$
 12.27 $\sin 4 \cosh 3 + i \cos 4 \sinh 3 = -7.62 - 6.55i, 10.05$
 12.28 $\tanh 1 = 0.762$ 12.29 1
 12.30 $-i$ 12.31 $(3+5i\sqrt{3})/8$
 12.32 $-4i/3$ 12.33 $i \tanh 1 = 0.762i$
 12.34 $i \sinh(\pi/2) = 2.301i$ 12.35 $-\cosh 2 = -3.76$
 12.36 $i \cosh 1 = 1.543i$ 12.37 $\cosh \pi$

Chapter 3

$$2.3 \quad \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \end{pmatrix}, \quad x = -3, y = 5$$

$$2.4 \quad \begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad x = (z + 1)/2, y = 1$$

$$2.5 \quad \begin{pmatrix} 1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{no solution}$$

$$2.6 \quad \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}, \quad x = 1, z = y$$

$$2.7 \quad \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}, \quad x = -4, y = 3$$

$$2.8 \quad \begin{pmatrix} 1 & -1 & 0 & -11 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad x = y - 11, z = 7$$

$$2.9 \quad \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{inconsistent, no solution}$$

$$2.10 \quad \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{inconsistent, no solution}$$

$$2.11 \quad \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{pmatrix}, \quad x = 2, y = -1, z = -3$$

$$2.12 \quad \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad x = -2, y = 1, z = 1$$

$$2.13 \quad \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -2 & 5/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad x = -2, y = 2z + 5/2$$

$$2.14 \quad \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{inconsistent, no solution}$$

6.8 $5x^2 + 3y^2 = 30$

6.9 $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad BA = \begin{pmatrix} 22 & 44 \\ -11 & -22 \end{pmatrix}$

6.10 $AC = AD = \begin{pmatrix} 11 & 12 \\ 33 & 36 \end{pmatrix}$

6.13 $\begin{pmatrix} 5/3 & -3 \\ -1 & 2 \end{pmatrix}$

6.14 $\frac{1}{6} \begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix}$

6.15 $-\frac{1}{2} \begin{pmatrix} 4 & 5 & 8 \\ -2 & -2 & -2 \\ 2 & 3 & 4 \end{pmatrix}$

6.16 $\frac{1}{8} \begin{pmatrix} -2 & 1 & 1 \\ 6 & -3 & 5 \\ 4 & 2 & 2 \end{pmatrix}$

6.17 $A^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 2 & -1 \\ 4 & 4 & -5 \\ 8 & 2 & -4 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$

$B^{-1}AB = \begin{pmatrix} 3 & 1 & 2 \\ -2 & -2 & -2 \\ -2 & -1 & 0 \end{pmatrix}$

$B^{-1}A^{-1}B = \frac{1}{6} \begin{pmatrix} 2 & 2 & -2 \\ -4 & -4 & -2 \\ 2 & -1 & 4 \end{pmatrix}$

6.19 $A^{-1} = \frac{1}{7} \begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}, \quad (x, y) = (5, 0)$

6.20 $A^{-1} = \frac{1}{7} \begin{pmatrix} -4 & 3 \\ 5 & -2 \end{pmatrix}, \quad (x, y) = (4, -3)$

6.21 $A^{-1} = \frac{1}{5} \begin{pmatrix} -1 & 2 & 2 \\ -2 & -1 & 4 \\ 3 & -1 & -1 \end{pmatrix}, \quad (x, y, z) = (-2, 1, 5)$

6.22 $A^{-1} = \frac{1}{12} \begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}, \quad (x, y, z) = (1, -1, 2)$

6.30 $\sin kA = A \sin k = \begin{pmatrix} 0 & \sin k \\ \sin k & 0 \end{pmatrix}, \quad \cos kA = I \cos k = \begin{pmatrix} \cos k & 0 \\ 0 & \cos k \end{pmatrix},$

$e^{kA} = \begin{pmatrix} \cosh k & \sinh k \\ \sinh k & \cosh k \end{pmatrix}, \quad e^{ikA} = \begin{pmatrix} \cos k & i \sin k \\ i \sin k & \cos k \end{pmatrix}$

6.32 $e^{i\theta B} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

In the following, L = linear, N = not linear.

7.1 N 7.2 L 7.3 N 7.4 L 7.5 L

7.6 N 7.7 L 7.8 N 7.9 N 7.10 N

7.11 N 7.12 L 7.13 (a) L (b) L

7.14 N 7.15 L 7.16 N 7.17 N

7.22 $D = 1$, rotation $\theta = -45^\circ$ 7.23 $D = 1$, rotation $\theta = 210^\circ$

7.24 $D = -1$, reflection line $x + y = 0$ 7.25 $D = -1$, reflection line $y = x\sqrt{2}$

7.26 $D = -1$, reflection line $x = 2y$ 7.27 $D = 1$, rotation $\theta = 135^\circ$

7.28 $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$

7.29 $\begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

7.30 $R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$; R is a 90° rotation about the z axis; S is a 90° rotation about the x axis.

7.31 From problem 30, $RS = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $SR = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$;
 RS is a 120° rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$; SR is a 120° rotation about $\mathbf{i} - \mathbf{j} + \mathbf{k}$.

7.32 180° rotation about $\mathbf{i} - \mathbf{k}$

7.33 120° rotation about $\mathbf{i} - \mathbf{j} - \mathbf{k}$

7.34 Reflection through the plane $y + z = 0$

7.35 Reflection through the (x, y) plane, and 90° rotation about the z axis.

8.1 In terms of basis $\mathbf{u} = \frac{1}{9}(9, 0, 7)$, $\mathbf{v} = \frac{1}{9}(0, -9, 13)$, the vectors are: $\mathbf{u} - 4\mathbf{v}$, $5\mathbf{u} - 2\mathbf{v}$, $2\mathbf{u} + \mathbf{v}$, $3\mathbf{u} + 6\mathbf{v}$.

8.2 In terms of basis $\mathbf{u} = \frac{1}{3}(3, 0, 5)$, $\mathbf{v} = \frac{1}{3}(0, 3, -2)$, the vectors are: $\mathbf{u} - 2\mathbf{v}$, $\mathbf{u} + \mathbf{v}$, $-2\mathbf{u} + \mathbf{v}$, $3\mathbf{u}$.

8.3 Basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

8.4 Basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

8.6 $\mathbf{V} = 3\mathbf{A} - \mathbf{B}$

8.7 $\mathbf{V} = \frac{3}{2}(1, -4) + \frac{1}{2}(5, 2)$

8.17 $x = 0, y = \frac{3}{2}z$

8.18 $x = -3y, z = 2y$

8.19 $x = y = z = w = 0$

8.20 $x = -z, y = z$

8.21 $\begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{pmatrix} = 0$

8.22 $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = 0$

8.23 For $\lambda = 3, x = 2y$; for $\lambda = 8, y = -2x$

8.24 For $\lambda = 7, x = 3y$; for $\lambda = -3, y = -3x$

8.25 For $\lambda = 2: x = 0, y = -3z$; for $\lambda = -3: x = -5y, z = 3y$;

for $\lambda = 4: z = 3y, x = 2y$

8.26 $\mathbf{r} = (3, 1, 0) + (-1, 1, 1)z$

8.27 $\mathbf{r} = (0, 1, 2) + (1, 1, 0)x$

8.28 $\mathbf{r} = (3, 1, 0) + (2, 1, 1)z$

9.3 $A^\dagger = \begin{pmatrix} 1 & 2i & 1 \\ 0 & 2 & 1 - i \\ -5i & 0 & 0 \end{pmatrix}$, $A^{-1} = \frac{1}{10} \begin{pmatrix} 0 & 5i - 5 & -10i \\ 0 & -5i & 10 \\ -2i & -1 - i & 2 \end{pmatrix}$

9.4 $A^\dagger = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, $A^{-1} = \frac{1}{6} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & i \\ -6 & 6i & -2 \end{pmatrix}$

9.14 $C^T B A^T, C^{-1} M^{-1} C, H$

10.1 (a) $d = 5$

(b) $d = 8$

(c) $d = \sqrt{56}$

10.2 The dimension of the space = the number of basis vectors listed.

One possible basis is given; other bases consist of the same number of independent linear combinations of the vectors given.

(a) $(1, -1, 0, 0), (-2, 0, 5, 1)$

(b) $(1, 0, 0, 5, 0, 1), (0, 1, 0, 0, 6, 4), (0, 0, 1, 0, -3, 0)$

(c) $(1, 0, 0, 0, -3), (0, 2, 0, 0, 1), (0, 0, 1, 0, -1), (0, 0, 0, 1, 4)$

$$11.29 \quad D = \begin{pmatrix} 11 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

$$11.30 \quad D = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}, \quad C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$11.31 \quad D = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$11.32 \quad D = \begin{pmatrix} 7 & 0 \\ 0 & 2 \end{pmatrix}, \quad C = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$11.41 \quad \lambda = 1, 3; \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$11.42 \quad \lambda = 1, 4; \quad U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ -1-i & 1 \end{pmatrix}$$

$$11.43 \quad \lambda = 2, -3; \quad U = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -i \\ -i & 2 \end{pmatrix}$$

$$11.44 \quad \lambda = 3, -7; \quad U = \frac{1}{5\sqrt{2}} \begin{pmatrix} 5 & -3-4i \\ 3-4i & 5 \end{pmatrix}$$

$$11.47 \quad U = \frac{1}{2} \begin{pmatrix} -1 & i\sqrt{2} & 1 \\ -1 & -i\sqrt{2} & 1 \\ \sqrt{2} & 0 & \sqrt{2} \end{pmatrix}$$

11.51 Reflection through the plane $3x - 2y - 3z = 0$, no rotation

11.52 60° rotation about $-\mathbf{i}\sqrt{2} + \mathbf{k}$ and reflection through the plane $z = x\sqrt{2}$

11.53 180° rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$

11.54 -120° (or 240°) rotation about $\mathbf{i}\sqrt{2} + \mathbf{j}$

11.55 Rotation -90° about $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, and reflection through the plane $x - 2y + 2z = 0$

11.56 45° rotation about $\mathbf{j} - \mathbf{k}$

$$11.58 \quad f(M) = \frac{1}{5} \begin{pmatrix} f(1) + 4f(6) & 2f(1) - 2f(6) \\ 2f(1) - 2f(6) & 4f(1) + f(6) \end{pmatrix}$$

$$M^4 = \frac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^4 & 2 - 2 \cdot 6^4 \\ 2 - 2 \cdot 6^4 & 4 + 6^4 \end{pmatrix} \quad M^{10} = \frac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^{10} & 2 - 2 \cdot 6^{10} \\ 2 - 2 \cdot 6^{10} & 4 + 6^{10} \end{pmatrix}$$

$$e^M = \frac{e}{5} \begin{pmatrix} 1 + 4e^5 & 2(1 - e^5) \\ 2(1 - e^5) & 4 + e^5 \end{pmatrix}$$

$$11.59 \quad M^4 = 2^3 \begin{pmatrix} 1 + 2^4 & 1 - 2^4 \\ 1 - 2^4 & 1 + 2^4 \end{pmatrix} \quad M^{10} = 2^3 \begin{pmatrix} 1 + 2^{10} & 1 - 2^{10} \\ 1 - 2^{10} & 1 + 2^{10} \end{pmatrix},$$

$$e^M = e^3 \begin{pmatrix} \cosh 1 & -\sinh 1 \\ -\sinh 1 & \cosh 1 \end{pmatrix}$$

$$12.2 \quad 3x'^2 - 2y'^2 = 24$$

$$12.3 \quad 10x'^2 = 35$$

$$12.4 \quad 5x'^2 - 5y'^2 = 8$$

$$12.5 \quad x'^2 + 3y'^2 + 6z'^2 = 14$$

$$12.6 \quad 3x'^2 + \sqrt{3}y'^2 - \sqrt{3}z'^2 = 12$$

$$12.7 \quad 3x'^2 + 5y'^2 - z'^2 = 60$$

$$12.14 \quad y = x \text{ with } \omega = \sqrt{k/m}; \quad y = -x \text{ with } \omega = \sqrt{5k/m}$$

$$12.15 \quad y = 2x \text{ with } \omega = \sqrt{3k/m}; \quad x = -2y \text{ with } \omega = \sqrt{8k/m}$$

$$12.16 \quad y = 2x \text{ with } \omega = \sqrt{2k/m}; \quad x = -2y \text{ with } \omega = \sqrt{7k/m}$$

$$12.17 \quad x = -2y \text{ with } \omega = \sqrt{2k/m}; \quad 3x = 2y \text{ with } \omega = \sqrt{2k/(3m)}$$

$$12.18 \quad y = x \text{ with } \omega = \sqrt{2k/m}; \quad x = -5y \text{ with } \omega = \sqrt{16k/(5m)}$$

$$12.19 \quad y = -x \text{ with } \omega = \sqrt{3k/m}; \quad y = 2x \text{ with } \omega = \sqrt{3k/(2m)}$$

$$12.21 \quad y = 2x \text{ with } \omega = \sqrt{k/m}; \quad x = -2y \text{ with } \omega = \sqrt{6k/m}$$

$$12.22 \quad y = -x \text{ with } \omega = \sqrt{2k/m}; \quad y = 3x \text{ with } \omega = \sqrt{2k/(3m)}$$

$$12.23 \quad y = -x \text{ with } \omega = \sqrt{k/m}; \quad y = 2x \text{ with } \omega = \sqrt{k/(4m)}$$

- 13.5 The 4's group 13.6 The cyclic group 13.7 The 4's group
 13.10 If $R = 90^\circ$ rotation, $P =$ reflection through the y axis, and $Q = PR$, then the 8 matrices of the symmetry group of the square are:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, R^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I,$$

$$R^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -R, P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, PR = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = Q,$$

$$PR^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -P, PR^3 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = -Q,$$

with multiplication table:

	I	R	-I	-R	P	Q	-P	-Q
I	I	R	-I	-R	P	Q	-P	-Q
R	R	-I	-R	I	-Q	P	Q	-P
-I	-I	-R	I	R	-P	-Q	P	Q
-R	-R	I	R	-I	Q	-P	-Q	P
P	P	Q	-P	-Q	I	R	-I	-R
Q	Q	-P	-Q	P	-R	I	R	-I
-P	-P	-Q	P	Q	-I	-R	I	R
-Q	-Q	P	Q	-P	R	-I	-R	I

- 13.11 The 4 matrices of the symmetry group of the rectangle are

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -P, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

This group is isomorphic to the 4's group.

13.14

Class	I	$\pm R$	-I	$\pm P$	$\pm Q$
Character	2	0	-2	0	0

- 13.20 Not a group (no unit element)
 13.21 $SO(2)$ is Abelian; $SO(3)$ is not Abelian.

For Problems 2 to 10, we list a possible basis.

- 14.2 $e^x, x e^x, e^{-x}$, or the three given functions
 14.3 $x, \cos x, x \cos x, e^x \cos x$
 14.4 $1, x, x^3$
 14.5 $1, x + x^3, x^2, x^4, x^5$
 14.6 Not a vector space
 14.7 $(1 + x^2 + x^4 + x^6), (x + x^3 + x^5 + x^7)$
 14.8 $1, x^2, x^4, x^6$
 14.9 Not a vector space; the negative of a vector with positive coefficients does not have positive coefficients.
 14.10 $(1 + \frac{1}{2}x), (x^2 + \frac{1}{2}x^3), (x^4 + \frac{1}{2}x^5), (x^6 + \frac{1}{2}x^7), (x^8 + \frac{1}{2}x^9),$
 $(x^{10} + \frac{1}{2}x^{11}), (x^{12} + \frac{1}{2}x^{13})$
 15.3 (a) $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z-2}{-2}$, $\mathbf{r} = (4, -1, 2) + (1, -2, -2)t$
 (b) $x - 5y + 3z = 0$ (c) $5/7$
 (d) $5\sqrt{2}/3$ (e) $\arcsin(19/21) = 64.8^\circ$
 15.4 (a) $4x + 2y + 5z = 10$ (b) $\arcsin(2/3) = 41.8^\circ$
 (c) $2/\sqrt{5}$ (d) $2x + y - 2z = 5$
 (e) $x = \frac{5}{2}, \frac{y}{2} = z, \mathbf{r} = \frac{5}{2}\mathbf{i} + (2\mathbf{j} + \mathbf{k})t$
 15.5 (a) $y = 7, \frac{x-2}{3} = \frac{z+1}{4}$, $\mathbf{r} = (2, 7, -1) + (3, 0, 4)t$
 (b) $x - 4y - 9z = 0$ (c) $\arcsin \frac{33}{35\sqrt{2}} = 41.8^\circ$
 (d) $\frac{12}{7\sqrt{2}}$ (e) $\frac{\sqrt{29}}{5}$

$$15.7 \quad A^T = \begin{pmatrix} 1 & 0 \\ -1 & i \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 1 & -i \\ 0 & -i \end{pmatrix} \quad AB = \begin{pmatrix} 2 & -2 & -6 \\ 0 & 3i & 5i \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 1 & -1 \\ 0 & -i \end{pmatrix} \quad B^T A^T = (AB)^T \quad B^T AC = \begin{pmatrix} 2 & 2 \\ 1 - 3i & 1 \\ -1 - 5i & -1 \end{pmatrix}$$

$$A^\dagger = \begin{pmatrix} 1 & 0 \\ -1 & -i \end{pmatrix} \quad B^T C = \begin{pmatrix} 0 & 2 \\ -3 & 1 \\ -5 & -1 \end{pmatrix} \quad C^{-1}A = \begin{pmatrix} 0 & -i \\ 1 & -1 \end{pmatrix}$$

$A^T B^T$, BA^T , ABC , $AB^T C$, $B^{-1}C$, and CB^T are meaningless.

$$15.8 \quad A^\dagger = \begin{pmatrix} 1 & -i & 1 \\ 0 & -3 & 0 \\ -2i & 0 & -i \end{pmatrix} \quad A^{-1} = \frac{1}{3i} \begin{pmatrix} -3i & 0 & 6i \\ 1 & -i & -2 \\ 3 & 0 & -3 \end{pmatrix}$$

$$15.9 \quad A = \begin{pmatrix} 1 + \frac{(n-1)d}{nR_2} & -(n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1 R_2} \right] \\ \frac{d}{n} & 1 - \frac{(n-1)d}{nR_1} \end{pmatrix}, \quad \frac{1}{f} = -A_{12}$$

$$15.10 \quad M = \begin{pmatrix} 1 - \frac{d}{f_2} & -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} \\ d & 1 - \frac{d}{f_1} \end{pmatrix}, \quad \frac{1}{f} = \frac{f_1 + f_2 - d}{f_1 f_2}, \quad \det M = 1$$

$$15.13 \quad \text{Area} = \frac{1}{2} |\vec{PQ} \times \vec{PR}| = 7/2$$

$$15.14 \quad x'' = -x, y'' = -y, \quad 180^\circ \text{ rotation}$$

$$15.15 \quad x'' = -y, y'' = x, \quad 90^\circ \text{ rotation of vectors or } -90^\circ \text{ rotation of axes}$$

$$15.16 \quad x'' = y, y'' = -x, z'' = z, \quad 90^\circ \text{ rotation of } (x, y) \text{ axes about the } z \text{ axis,}$$

or -90° rotation of vectors about the z axis

$$15.17 \quad x'' = x, y'' = -y, z'' = -z, \quad \text{rotation of } \pi \text{ about the } x \text{ axis}$$

$$15.18 \quad \begin{matrix} 1 & (1, 1) \\ -2 & (0, 1) \end{matrix} \quad 15.19 \quad \begin{matrix} 6 & (1, 1) \\ 1 & (1, -4) \end{matrix} \quad 15.20 \quad \begin{matrix} 1 & (1, 1) \\ 9 & (1, -1) \end{matrix}$$

$$15.21 \quad \begin{matrix} 0 & (1, -2) \\ 5 & (2, 1) \end{matrix} \quad 15.22 \quad \begin{matrix} 1 & (1, 0, 1) \\ 4 & (0, 1, 0) \\ 5 & (1, 0, -1) \end{matrix} \quad 15.23 \quad \begin{matrix} 1 & (1, 1, -2) \\ 3 & (1, -1, 0) \\ 4 & (1, 1, 1) \end{matrix}$$

$$15.24 \quad \begin{matrix} 2 & (0, 4, 3) \\ 7 & (5, -3, 4) \\ -3 & (5, 3, -4) \end{matrix}$$

$$15.25 \quad C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 1 & \sqrt{2} \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} \sqrt{2} & 0 \\ -1 & 1 \end{pmatrix}$$

$$15.26 \quad C = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{17} \\ 1/\sqrt{2} & -4/\sqrt{17} \end{pmatrix}, \quad C^{-1} = \frac{1}{5} \begin{pmatrix} 4\sqrt{2} & \sqrt{2} \\ \sqrt{17} & -\sqrt{17} \end{pmatrix}$$

$$15.27 \quad 3x'^2 - y'^2 - 5z'^2 = 15, \quad d = \sqrt{5}$$

$$15.28 \quad 9x'^2 + 4y'^2 - z'^2 = 36, \quad d = 2$$

$$15.29 \quad 3x'^2 + 6y'^2 - 4z'^2 = 54, \quad d = 3$$

$$15.30 \quad 7x'^2 + 20y'^2 - 6z'^2 = 20, \quad d = 1$$

$$15.31 \quad \omega = (k/m)^{1/2}, \quad (7k/m)^{1/2}$$

$$15.32 \quad \omega = 2(k/m)^{1/2}, \quad (3k/m)^{1/2}$$

Chapter 4

- 1.1 $\partial u/\partial x = 2xy^2/(x^2 + y^2)^2$, $\partial u/\partial y = -2x^2y/(x^2 + y^2)^2$
 1.2 $\partial s/\partial t = ut^{u-1}$, $\partial s/\partial u = t^u \ln t$
 1.3 $\partial z/\partial u = u/(u^2 + v^2 + w^2)$
 1.4 At (0, 0), both = 0; at (-2/3, 2/3), both = -4
 1.5 At (0, 0), both = 0; at (1/4, $\pm 1/2$), $\partial^2 w/\partial x^2 = 6$, $\partial^2 w/\partial y^2 = 2$
 1.7 $2x$ 1.8 $-2x$ 1.9 $2x(1 + 2 \tan^2 \theta)$
 1.10 $4y$ 1.11 $2y$ 1.12 $2y(\cot^2 \theta + 2)$
 1.13 $4r^2 \tan \theta$ 1.14 $-2r^2 \cot \theta$ 1.15 $r^2 \sin 2\theta$
 1.16 $2r(1 + \sin^2 \theta)$ 1.17 $4r$ 1.18 $2r$
 1.19 0 1.20 $8y \sec^2 \theta$ 1.21 $-4x \csc^2 \theta$
 1.22 0 1.23 $2r \sin 2\theta$ 1.24 0
 1.7' $-2y^4/x^3$ 1.8' $-2r^4/x^3$ 1.9' $2x \tan^2 \theta \sec^2 \theta$
 1.10' $2y + 4y^3/x^2$ 1.11' $2yr^4/(r^2 - y^2)^2$ 1.12' $2y \sec^2 \theta$
 1.13' $2x^2 \sec^2 \theta \tan \theta (\sec^2 \theta + \tan^2 \theta)$
 1.14' $2y^2 \sec^2 \theta \tan \theta$ 1.15' $2r^2 \tan \theta \sec^2 \theta$ 1.16' $2r \tan^2 \theta$
 1.17' $4r^3/x^2 - 2r$ 1.18' $-2ry^4/(r^2 - y^2)^2$
 1.19' $-8r^3y^3/(r^2 - y^2)^3$ 1.20' $4x \tan \theta \sec^2 \theta (\tan^2 \theta + \sec^2 \theta)$
 1.21' $4y \sec^2 \theta \tan \theta$ 1.22' $-8r^3/x^3$
 1.23' $4r \tan \theta \sec^2 \theta$ 1.24' $-8y^3/x^3$
- 2.1 $y + y^3/6 - x^2y/2 + x^4y/24 - x^2y^3/12 + y^5/120 + \dots$
 2.2 $1 - (x^2 + 2xy + y^2)/2 + (x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4)/24 + \dots$
 2.3 $x - x^2/2 - xy + x^3/3 + x^2y/2 + xy^2 \dots$
 2.4 $1 + xy + x^2y^2/2 + x^3y^3/3! + x^4y^4/4! \dots$
 2.5 $1 + \frac{1}{2}xy - \frac{1}{8}x^2y^2 + \frac{1}{16}x^3y^3 - \frac{5}{128}x^4y^4 \dots$
 2.6 $1 + x + y + (x^2 + 2xy + y^2)/2 \dots$
 2.8 $e^x \cos y = 1 + x + (x^2 - y^2)/2 + (x^3 - 3xy^2)/3! \dots$
 $e^x \sin y = y + xy + (3x^2y - y^3)/3! \dots$
- 4.2 2.5×10^{-13} 4.3 14.8 4.4 12.2 4.5 14.96
 4.6 9% 4.7 15% 4.8 5% 4.10 4.28 nt
 4.11 3.95 4.12 2.01 4.13 5/3 4.14 0.005
 4.15 8×10^{23}
- 5.1 $e^{-y} \sinh t + z \sin t$ 5.2 $w = 1$, $dw/dp = 0$
 5.3 $2r(q^2 - p^2)$ 5.4 $(4ut + 2v \sin t)/(u^2 - v^2)$
 5.6 $5(x + y)^4(1 + 10 \cos 10x)$ 5.7 $(1 - 2b - e^{2a}) \cos(a - b)$
- 6.1 $dv/dp = -v/(ap)$, $d^2v/dp^2 = v(1 + a)/(a^2p^2)$
 6.2 $y' = 1$, $y'' = 0$ 6.3 $y' = 4(\ln 2 - 1)/(2 \ln 2 - 1)$

- 13.8 $\pi^{-1}\text{ft} \cong 4 \text{ inches}$
13.9 $dz/dt = 1 + t(2 - x - y)/z, z \neq 0$
13.10 $(x \ln x - y^2/x)x^y$ where $x = r \cos \theta, y = r \sin \theta$
13.11 $\frac{dy}{dx} = -\frac{b^2x}{a^2y}, \frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$
13.12 13
13.13 -1
13.14 $(\partial w/\partial x)_y = (\partial f/\partial x)_{s, t} + 2(\partial f/\partial s)_{x, t} + 2(\partial f/\partial t)_{x, s} = f_1 + 2f_2 + 2f_3$
13.15 $(\partial w/\partial x)_y = f_1 + 2xf_2 + 2yf_3$
13.17 $\sqrt{19}$
13.18 $\sqrt{26}/3$
13.19 $1/27$
13.20 At $x = -1, y = 20$; at $x = 1/2, y = -1/4$
13.21 $T(2) = 4, T(5) = -5$
13.22 $T(5, 0) = 10, T(2, \pm\sqrt{2}) = -4$
13.23 $t \cot t$
13.24 0
13.25 $-e^x/x$
13.26 $3 \sin x^3/x$
13.29 $dt = 3.9$
13.30 $2f(x, x) + \int_0^x \frac{\partial}{\partial x} [f(x, u) + f(u, x)] du$

Chapter 5

- 2.1 3 2.2 -18 2.3 4 2.4 $8/3$
 2.5 $\frac{e^2}{4} - \frac{5}{12}$ 2.6 2.35 2.7 $5/3$ 2.8 $1/2$
 2.9 6 2.10 5π 2.11 36 2.12 2
 2.13 $7/4$ 2.14 $4 - e(\ln 4)$ 2.15 $3/2$ 2.16 $(\ln 3)/6$
 2.17 $(\ln 2)/2$ 2.18 $(8\sqrt{2} - 7)/3$ 2.19 32 2.20 16
 2.21 $131/6$ 2.22 $5/3$ 2.23 $9/8$ 2.24 $9/2$
 2.25 $3/2$ 2.26 $4/3$ 2.27 $32/5$ 2.28 $1/3$
 2.29 2 2.30 $1 - e^{-2}$ 2.31 6 2.32 $e - 1$
 2.33 $16/3$ 2.34 $8192k/15$ 2.35 $216k$ 2.36 $1/6$
 2.37 $7/6$ 2.38 -20 2.39 70 2.40 $3/2$
 2.41 5 2.42 4 2.43 $9/2$ 2.44 $7k/3$
 2.45 $46k/15$ 2.46 $8k$ 2.47 $16/3$ 2.48 $16\pi/3$
 2.49 $1/3$ 2.50 $64/3$
- 3.2 (a) ρl (b) $Ml^2/12$ (c) $Ml^2/3$
 3.3 (a) $M = 140$ (b) $\bar{x} = 130/21$ (c) $I_m = 6.92M$ (d) $I = 150M/7$
 3.4 (a) $M = 3l/2$ (b) $\bar{x} = 4l/9$ (c) $I_m = \frac{13}{162}Ml^2$ (d) $I = 5Ml^2/18$
 3.5 (a) $Ma^2/3$ (b) $Ma^2/12$ (c) $2Ma^2/3$
 3.6 (a) (2, 2) (b) $6M$ (c) $2M$
 3.7 (a) $M = 9$ (b) $(\bar{x}, \bar{y}) = (2, 4/3)$
 (c) $I_x = 2M, I_y = 9M/2$ (d) $I_m = 13M/18$
- 3.8 $2Ma^2/3$
 3.9 (a) $1/6$ (b) $(1/4, 1/4, 1/4)$ (c) $M = 1/24, \bar{z} = 2/5$
 3.10 (a) $s = 2 \sinh 1$ (b) $\bar{y} = (2 + \sinh 2)/(4 \sinh 1) = 1.2$
 3.11 (a) $M = (5\sqrt{5} - 1)/6 = 1.7$
 (b) $\bar{x} = 0, M\bar{y} = (25\sqrt{5} + 1)/60 = 0.95, \bar{y} = (313 + 15\sqrt{5})/620 = 0.56$
- 3.14 $V = 2\pi^2 a^2 b, A = 4\pi^2 ab$, where a = radius of revolving circle,
 b = distance to axis from center of this circle.
- 3.15 For area, $(\bar{x}, \bar{y}) = (0, \frac{4}{3}r/\pi)$; for arc, $(\bar{x}, \bar{y}) = (0, 2r/\pi)$
- 3.17 $4\sqrt{2}/3$ 3.18 $s = [3\sqrt{2} + \ln(1 + \sqrt{2})]/2 = 2.56$
 3.19 2π 3.20 $13\pi/3$
- 3.21 $s\bar{x} = [51\sqrt{2} - \ln(1 + \sqrt{2})]/32 = 2.23, s\bar{y} = 13/6, s$ as in Problem 3.18;
 then $\bar{x} = 0.87, \bar{y} = 0.85$
- 3.22 $(4/3, 0, 0)$
 3.23 $(149/130, 0, 0)$
 3.24 $2M/5$
 3.25 I/M has the same numerical value as \bar{x} in 3.21.
 3.26 $2M/3$ 3.27 $\frac{149}{130}M$ 3.28 $13/6$ 3.29 2 3.30 $32/5$

- 4.1 (b) $\bar{x} = \bar{y} = 4a/(3\pi)$
(c) $I = Ma^2/4$
(e) $\bar{x} = \bar{y} = 2a/\pi$
- 4.2 (c) $\bar{y} = 4a/(3\pi)$
(d) $I_x = Ma^2/4, I_y = 5Ma^2/4, I_z = 3Ma^2/2$
(e) $\bar{y} = 2a/\pi$
(f) $\bar{x} = 6a/5, I_x = 48Ma^2/175, I_y = 288Ma^2/175, I_z = 48Ma^2/25$
(g) $A = (\frac{2}{3}\pi - \frac{1}{2}\sqrt{3})a^2$
- 4.3 (a), (b), or (c) $\frac{1}{2}Ma^2$
- 4.4 (a) $4\pi a^2$ (b) $(0, 0, a/2)$ (c) $2Ma^2/3$
(d) $4\pi a^3/3$ (e) $(0, 0, 3a/8)$
- 4.5 $7\pi/3$ 4.6 $\pi \ln 2$
- 4.7 (a) $V = 2\pi a^3(1 - \cos \alpha)/3$ (b) $\bar{z} = 3a(1 + \cos \alpha)/8$
- 4.8 $I_z = Ma^2/4$
- 4.10 (a) $V = 64\pi$ (b) $\bar{z} = 231/64$
- 4.11 12π
- 4.12 (c) $M = (16\rho/9)(3\pi - 4) = 9.64\rho$
 $I = (128\rho/15^2)(15\pi - 26) = 12.02\rho = 1.25M$
- 4.13 (b) $\pi a^2(z_2 - z_1) - \pi(z_2^3 - z_1^3)/3$ (c) $\frac{\frac{1}{2}a^2(z_2^2 - z_1^2) - \frac{1}{4}(z_2^4 - z_1^4)}{a^2(z_2 - z_1) - (z_2^3 - z_1^3)/3}$
- 4.14 $\pi(1 - e^{-1})/4$ 4.16 $u^2 + v^2$
- 4.17 $a^2(\sinh^2 u + \sin^2 v)$ 4.19 $\pi/4$
- 4.20 $1/12$ 4.22 $12(1 + 36\pi^2)^{1/2}$
- 4.23 Length = $(R \sec \alpha)$ times change in latitude
- 4.24 $\rho G \pi a/2$
- 4.26 (a) $7Ma^2/5$ (b) $3Ma^2/2$
- 4.27 $2\pi ah$ (where h = distance between parallel planes)
- 4.28 $(0, 0, a/2)$
- 5.1 $9\pi\sqrt{30}/5$ 5.2 $\pi\sqrt{7/5}$
- 5.3 $\pi(37^{3/2} - 1)/6 = 117.3$ 5.4 $\pi/\sqrt{6}$
- 5.5 8π for each nappe 5.6 4
- 5.7 4 5.8 $[3\sqrt{6} + 9 \ln(\sqrt{2} + \sqrt{3})] / 16$
- 5.9 $\pi\sqrt{2}$ 5.10 $2\pi a^2(\sqrt{2} - 1)$
- 5.11 $(\bar{x}, \bar{y}, \bar{z}) = (1/3, 1/3, 1/3)$ 5.12 $M = \sqrt{3}/6, (\bar{x}, \bar{y}, \bar{z}) = (1/2, 1/4, 1/4)$
- 5.13 $\bar{z} = \frac{\pi}{4(\pi - 2)}$ 5.14 $M = \frac{\pi}{2} - \frac{4}{3}$
- 5.15 $I_z/M = \frac{2(3\pi - 7)}{9(\pi - 2)} = 0.472$ 5.16 $\bar{x} = 0, \bar{y} = 1, \bar{z} = \frac{32}{9\pi}\sqrt{\frac{2}{5}} = 0.716$
- 6.1 $7\pi(2 - \sqrt{2})/3$ 6.2 $45(2 + \sqrt{2})/112$ 6.3 $15\pi/8$
- 6.4 (a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
- 6.5 cone: $2\pi ab^2/3$; ellipsoid: $4\pi ab^2/3$; cylinder: $2\pi ab^2$
- 6.6 (a) $\frac{4\pi - 3\sqrt{3}}{6}$ (b) $\bar{x} = \frac{5}{4\pi - 3\sqrt{3}}, \bar{y} = \frac{6\sqrt{3}}{4\pi - 3\sqrt{3}}$
- 6.7 $\frac{8\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}M$ 6.8 (a) $5\pi/3$ (b) $27/20$
- 6.9 $(\bar{x}, \bar{y}) = (0, 3c/5)$
- 6.10 (a) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ (b) $\pi^2/2$ (c) $3M/8$
- 6.11 $\bar{z} = 3h/4$ 6.12 $(abc)^2/6$
- 6.13 $8a^2$ 6.14 $16a^3/3$
- 6.15 $I_x = 8Ma^2/15, I_y = 7Ma^2/15$ 6.16 $\bar{x} = \bar{y} = 2a/5$

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