SIXTH EDITION

Introduction to FLUID MECHANICS

A NEW book contains CD-based resources for enhanced learning!

Robert W. Fox
Alan T. McDonald
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Table G.1 SI Units and Prefixes[a]

SI Units	Quantity	Unit	SI Symbol	Formula
SI base units:	Length	meter	m	—
	Mass	kilogram	kg	—
	Time	second	s	—
	Temperature	kelvin	K	—
SI supplementary unit:	Plane angle	radian	rad	—
SI derived units:	Energy	joule	J	N · m
	Force	newton	N	$kg \cdot m/s^2$
	Power	watt	W	J/s
	Pressure	pascal	Pa	N/m^2
	Work	joule	J	N · m

SI prefixes	Multiplication Factor	Prefix	SI Symbol
	$1\,000\,000\,000\,000 = 10^{12}$	tera	T
	$1\,000\,000\,000 = 10^{9}$	giga	G
	$1\,000\,000 = 10^{6}$	mega	M
	$1\,000 = 10^{3}$	kilo	k
	$0.01 = 10^{-2}$	centi[b]	c
	$0.001 = 10^{-3}$	milli	m
	$0.000\,001 = 10^{-6}$	micro	μ
	$0.000\,000\,001 = 10^{-9}$	nano	n
	$0.000\,000\,000\,001 = 10^{-12}$	pico	p

[a] Source: ASTM Standard for Metric Practice E 380-97, 1997.
[b] To be avoided where possible.

Table G.2 Conversion Factors and Definitions

Fundamental Dimension	English Unit	Exact SI Value	Approximate SI Value
Length	1 in.	0.0254 m	—
Mass	1 lbm	0.453 592 37 kg	0.454 kg
Temperature	1°F	5/9 K	—

Definitions:

Acceleration of gravity:	$g = 9.8066$ m/s^2 (= 32.174 ft/s^2)
Energy:	Btu (British thermal unit) = amount of energy required to raise the temperature of 1 lbm of water 1°F (1 Btu = 778.2 ft · lbf)
	kilocalorie = amount of energy required to raise the temperature of 1 kg of water 1 K (1 kcal = 4187 J)
Length:	1 mile = 5280 ft; 1 nautical mile = 6076.1 ft = 1852 m (exact)
Power:	1 horsepower = 550 ft · lbf/s
Pressure:	1 bar = 10^5 Pa
Temperature:	degree Fahrenheit, $T_F = \frac{9}{5}T_C + 32$ (where T_C is degrees Celsius)
	degree Rankine, $T_R = T_F + 459.67$
	Kelvin, $T_K = T_C + 273.15$ (exact)
Viscosity:	1 Poise = 0.1 kg/(m · s)
	1 Stoke = 0.0001 m^2/s
Volume:	1 gal = 231 in.^{3} (1 ft^3 = 7.48 gal)

Useful Conversion Factors:

1 lbf = 4.448 N

1 lbf/in.^{2} = 6895 Pa

1 Btu = 1055 J

1 hp = 746 W = 2545 Btu/hr

1 kW = 3413 Btu/hr

1 quart = 0.000946 m^3 = 0.946 liter

1 kcal = 3.968 Btu

INTRODUCTION TO FLUID MECHANICS

SIXTH EDITION

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JOHN WILEY & SONS, INC.

On the Cover:

Aerodynamics in action at speeds of 200+ miles per hour!

The cover photo shows the Formula 1 Ferrari cars of World Driving Champion Michael Schumacher and his teammate Rubens Barrichello at the United States Grand Prix on September 29, 2002. The location is the road circuit of the Indianapolis Motor Speedway in Indianapolis, Indiana. Similar Ferrari 1-2 finishes were seen at many races during 2002.

All modern racing cars use aerodynamic downforce (negative lift) to improve traction without adding significant weight to the car. Using high downforce allows high cornering speeds on the twisting, turning road courses typical of Formula 1 races. The maximum downforce can exceed twice the weight of the car at 200+ miles per hour straightaway speeds! Of course high downforce also causes high drag, which reduces straightaway speed, so a compromise is needed.

The photo clearly shows some features of Schumacher's Ferrari. Notable is the extensive use of aerodynamic devices designed to develop and control downforce.

The Ferrari's front wings are two-element designs. They are made as large, and placed as far forward on the chassis, as the rules allow. The rear wing appears to be a three-element design. The rear wing also is made as large as the rules allow; it is placed as far rearward on the chassis as possible. The side-mounted engine-cooling radiators are located in housings faired smoothly on the outside to minimize drag. The radiator housings are designed with careful flow management inside to maximize the flow of cooling air. Also visible are fairings to direct hot air from the radiators around the rear tires, and at the front of the car, cool air toward the brakes.

Details of underbody airflow management, commonly called "ground effects," are not so easily seen. Airflow under the car is routed carefully, using diffusers designed to the limits of the rules, to develop the most negative pressure, and cause it to act over the largest possible area under the car, to develop additional downforce.

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PREFACE

This text was written for an introductory course in fluid mechanics. Our approach to the subject, as in previous editions, emphasizes the physical concepts of fluid mechanics and methods of analysis that begin from basic principles. The primary objective of this book is to help users develop an orderly approach to problem solving. Thus we always start from governing equations, state assumptions clearly, and try to relate mathematical results to corresponding physical behavior. We emphasize the use of control volumes to maintain a practical problem-solving approach that is also theoretically inclusive.

This approach is illustrated by 116 example problems in the text. Solutions to the example problems have been prepared to illustrate good solution technique and to explain difficult points of theory. Example problems are set apart in format from the text so they are easy to identify and follow. Forty-five example problems include *Excel* workbooks on the accompanying CD-ROM, making them useful for "What if?" analyses by students or by the instructor during class.

Additional important information about the text and our procedures is given in the "Note to Students" section on page 1 of the printed text. We urge you to study this section carefully and to integrate the suggested procedures into your problem solving and results-presentation approaches.

SI units are used in about 70 percent of both example and end-of-chapter problems. English Engineering units are retained in the remaining problems to provide experience with this traditional system and to highlight conversions among unit systems that may be derived from fundamentals.

Complete explanations presented in the text, together with numerous detailed examples, make this book understandable for students. This frees the instructor to depart from conventional lecture teaching methods. Classroom time can be used to bring in outside material, expand upon special topics (such as non-Newtonian flow, boundary-layer flow, lift and drag, or experimental methods), solve example problems, or explain difficult points of assigned homework problems. In addition, the 45 example problem *Excel* workbooks are useful for presenting a variety of fluid mechanics phenomena, especially the effects produced when varying input parameters. Thus each class period can be used in the manner most appropriate to meet student needs.

The material has been selected carefully to include a broad range of topics suitable for a one- or two-semester course at the junior or senior level. We assume a background in rigid body dynamics and mathematics through differential equations. A background in thermodynamics is desirable for studying compressible flow.

More advanced material, not typically covered in a first course, has been moved to the CD. There the advanced material is available to interested users of the book; on the CD it does not interrupt the topic flow of the printed text.

Material in the printed text has been organized into broad topic areas:

- Introductory concepts, scope of fluid mechanics, and fluid statics (Chapters 1, 2, and 3)
- Development and application of control volume forms of basic equations (Chapter 4)
- Development and application of differential forms of basic equations (Chapters 5 and 6)
- Dimensional analysis and correlation of experimental data (Chapter 7).
- Applications for internal viscous incompressible flows (Chapter 8).
- Applications for external viscous incompressible flows (Chapter 9).
- Analysis of fluid machinery and system applications (Chapter 10)
- Analysis and applications of one- and two-dimensional compressible flows (Chapters 11 and 12).

Chapter 4 deals with analysis using both finite and differential control volumes. The Bernoulli equation is derived (in an optional sub-section of Section 4-4) as an example application of the basic equations to a differential control volume. Being able to use the Bernoulli equation in Chapter 4 allows us to include more challenging problems dealing with the momentum equation for finite control volumes.

Another derivation of the Bernoulli equation is presented in Chapter 6, where it is obtained by integrating Euler's equation along a streamline. If an instructor chooses to delay introducing the Bernoulli equation, the challenging problems from Chapter 4 may be assigned during study of Chapter 6.

This edition incorporates a number of significant changes. In Chapter 7, the discussion of non-dimensionalizing the governing equations to obtain dimensionless parameters is moved to the beginning of the chapter. Chapter 8 incorporates pumps into the discussion of energy considerations in pipe flow. The discussion of multiple-path pipe systems is expanded and illustrated with an interactive *Excel* workbook. Chapter 10 has been restructured to include separate sub-topics on machines for doing work on, and machines for extracting work from, a fluid. Chapter 12 has been completely restructured so that the basic equations for one-dimensional compressible flow are derived once, and then applied to each special case (isentropic flow, nozzle flow, Fanno line flow, Rayleigh line flow, and normal shocks). Finally, a new section on oblique shocks and expansion waves is included.

We have made a major effort to improve clarity of writing in this edition. Professor Philip J. Pritchard of Manhattan College, has joined the Fox-McDonald team as co-author. Professor Pritchard reviewed the entire manuscript in detail to clarify and improve discussions, added numerous physical examples, and prepared the *Excel* workbooks that accompany 45 example problems and over 300 end-of chapter problems. His contributions have been extraordinary.

The sixth edition includes 1315 end-of-chapter problems. Many problems have been combined and contain multiple parts. Most have been structured so that all parts need not be assigned at once, and almost 25 percent of sub-parts have been designed to explore "What if?" questions.

About 300 problems are new or modified for this edition, and many include a component best suited for analysis using a spreadsheet. A CD icon in the margin identifies these problems. Many of these problems have been designed so the computer component provides a parametric investigation of a single-point solution, to facilitate and encourage students in their attempts to perform "What if?" experimentation. The *Excel* workbooks prepared by Professor Pritchard aid this process significantly. A new Appendix H, "A Brief Review of Microsoft *Excel*," also has been added to the CD.

We have included many open-ended problems. Some are thought-provoking questions intended to test understanding of fundamental concepts, and some require creative thought, synthesis, and/or narrative discussion. We hope these problems will inspire each instructor to develop and use more open-ended problems.

The Solutions Manual for the sixth edition continues the tradition established by Fox and McDonald: it contains a complete, detailed solution for each of the 1315 homework problems. Each solution is prepared in the same systematic way as the example problem solutions in the printed text. Each solution begins from governing equations, clearly states assumptions, reduces governing equations to computing equations, obtains an algebraic result, and finally substitutes numerical values to calculate a quantitative answer. Solutions may be reproduced for classroom or library use, eliminating the labor of problem solving for the instructor who adopts the text.

Problems in each chapter are arranged by topic, and within each topic they generally increase in complexity or difficulty. This makes it easy for the instructor to assign homework problems at the appropriate difficulty level for each section of the book. The Solutions Manual is available in CD form directly from the publisher upon request after the text is adopted. Go to the text's website at www.wiley.com/college/fox to request access to the password-protected online version, or to www.wiley.com/college to find your local Wiley representative and request the Solutions Manual in CD form.

Where appropriate, we have used open-ended design problems in place of traditional laboratory experiments. For those who do not have complete laboratory facilities, students could be assigned to work in teams to solve these problems. Design problems encourage students to spend more time exploring applications of fluid mechanics principles to the design of devices and systems. In the sixth edition, design problems are included with the end-of-chapter problems.

The presentation of flow functions for compressible flow in Appendix E has been expanded to include data for oblique shocks and expansion waves. Expanded forms of each table in this appendix can be printed from the associated *Excel* workbooks, including tables for ideal gases other than air.

Many worthwhile videos are available to demonstrate and clarify basic principles of fluid mechanics. These are referenced in the text where their use would be appropriate and are also identified by supplier in Appendix C.

When students finish the fluid mechanics course, we expect them to be able to apply the governing equations to a variety of problems, including those they have not encountered previously. In the sixth edition we particularly emphasize physical concepts throughout to help students model the variety of phenomena that occur in real fluid flow situations. We minimize use of "magic formulas" and emphasize the systematic and fundamental approach to problem solving. By following this format, we believe students develop confidence in their ability to apply the material and find they can reason out solutions to rather challenging problems.

The book is well suited for independent study by students or practicing engineers. Its readability and clear examples help to build confidence. Answers to many quantitative problems are provided at the back of the printed text.

We recognize that no single approach can satisfy all needs, and we are grateful to the many students and faculty whose comments have helped us improve upon earlier editions of this book. We especially thank our reviewers for the sixth edition: Mark A. Cappelli of Stanford University, Edward M. Gates of California State Polytechnic University (Pomona), Jay M. Khodadadi of Auburn University, Tim Lee of

McGill University, and S. A. Sherif of University of Florida. We look forward to continued interactions with these and other colleagues who use the book.

We appreciate the unstinting support of our wives, Beryl, Tania, and Penelope. They are keenly aware of all the hours that went into this effort!

We welcome suggestions and/or criticisms from interested users of this book.

Robert W. Fox
Alan T. McDonald
Philip J. Pritchard
April 2003

CONTENTS

Chapter 1

INTRODUCTION

The goal of this textbook is to provide a clear, concise introduction to the subject of fluid mechanics. In beginning the study of any subject, a number of questions may come to mind. Students in the first course in fluid mechanics might ask:

What is fluid mechanics all about?

Why do I have to study it?

Why should I want to study it?

How does it relate to subject areas with which I am already familiar?

In this chapter we shall try to present some answers to these and similar questions. This should serve to establish a base and a perspective for our study of fluid mechanics. Before proceeding with the definition of a fluid, we digress for a moment with a few comments to students.

1-1 NOTE TO STUDENTS

In writing this book we have kept you, the student, uppermost in our minds; the book is written for you. It is our strong feeling that classroom time should *not* be devoted to a regurgitation of textbook material by the instructor. Instead, the time should be used to amplify the textbook material by discussing related material and applying basic principles to the solution of problems. This requires (1) a clear, concise presentation of the fundamentals that you, the student, can read and understand, and (2) your willingness to read the text before going to class. We have assumed responsibility for meeting the first requirement. You must assume responsibility for satisfying the second requirement. There probably will be times when we fall short of these objectives. If so, we would appreciate hearing from you either directly (at philip.pritchard@manhattan.edu) or through your instructor.

It goes without saying that an introductory text is not all-inclusive. Your instructor undoubtedly will expand on the material presented, suggest alternative approaches to topics, and introduce additional new material. We encourage you to refer to the many other fluid mechanics textbooks and references available in the library and on the Web; where another text presents a particularly good discussion of a given topic, we shall refer to it directly.

We also encourage you to learn from your fellow students and from the graduate assistant(s) assigned to the course as well as from your instructor. We assume that you have had an introduction to thermodynamics (either in a basic physics course or an introductory course in thermodynamics) and prior courses in statics, dynamics, and differential and integral calculus. No attempt will be made to restate this subject material; however, the pertinent aspects of this previous study will be reviewed briefly when appropriate.

It is our strong belief that one learns best by *doing*. This is true whether the subject under study is fluid mechanics, thermodynamics, or golf. The fundamentals in any of these are few, and mastery of them comes through practice. *Thus it is extremely important that you solve problems.* The numerous problems included at the end of each chapter provide the opportunity to practice applying fundamentals to the solution of problems. You should avoid the temptation to adopt a "plug and chug" approach to solving problems. Most of the problems are such that this approach simply will not work. In solving problems we strongly recommend that you proceed using the following logical steps:

1. State briefly and concisely (in your own words) the information given.
2. State the information to be found.
3. Draw a schematic of the system or control volume to be used in the analysis. Be sure to label the boundaries of the system or control volume and label appropriate coordinate directions.
4. Give the appropriate mathematical formulation of the *basic* laws that you consider necessary to solve the problem.
5. List the simplifying assumptions that you feel are appropriate in the problem.
6. Complete the analysis algebraically before substituting numerical values.
7. Substitute numerical values (using a consistent set of units) to obtain a numerical answer.
   a. Reference the source of values for any physical properties.
   b. Be sure the significant figures in the answer are consistent with the given data.
8. Check the answer and review the assumptions made in the solution to make sure they are reasonable.
9. Label the answer.

In your initial work this problem format may seem unnecessary and even long-winded. However, such an orderly approach to the solution of problems will reduce errors, save time, and permit a clearer understanding of the limitations of a particular solution. This approach also prepares you for communicating your solution method and results to others, as will often be necessary in your career. *This format is used in all example problems presented in this text*; answers to example problems are rounded to three significant figures.

Most engineering calculations involve measured values or physical property data. Every measured value has associated with it an experimental uncertainty. The uncertainty in a measurement can be reduced with care and by applying more precise measurement techniques, but cost and time needed to obtain data rise sharply as measurement precision is increased. Consequently, few engineering data are sufficiently precise to justify the use of more than three significant figures.

Not all measurements can be made to the same degree of accuracy and not all data are equally good; the validity of data should be documented before test results are used for design. A statement of the probable uncertainty of data is an important part of reporting experimental results completely and clearly. Analysis of uncertainty also is useful during experiment design. Careful study may indicate potential sources of unacceptable error and suggest improved measurement methods.

The principles of specifying the experimental uncertainty of a measurement and of estimating the uncertainty of a calculated result are reviewed in Appendix F.

These should be understood thoroughly by anyone who performs laboratory work. We suggest you take time to review Appendix F before performing laboratory work or solving the homework problems at the end of this chapter.

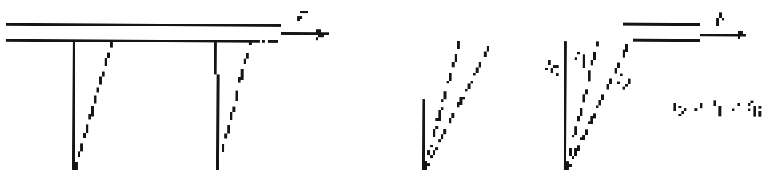
1-2 DEFINITION OF A FLUID

We already have a common sense idea of when we are working with a fluid, as opposed to a solid: Fluids tend to flow when we interact with them (e.g., when you stir your morning coffee); solids tend to deform or bend (e.g., when you type on a keyboard, the springs under the keys compress). Engineers need a more formal and precise definition of a fluid: A *fluid* is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.

Thus fluids comprise the liquid and gas (or vapor) phases of the physical forms in which matter exists. The distinction between a fluid and the solid state of matter is clear if you compare fluid and solid behavior. A solid deforms when a shear stress is applied, but its deformation does not continue to increase with time.

In Fig. 1.1 the deformations of a solid (Fig. 1.1*a*) and a fluid (Fig. 1.1*b*) under the action of a constant shear force are contrasted. In Fig. 1.1*a* the shear force is applied to the solid through the upper of two plates to which the solid has been bonded. When the shear force is applied to the plate, the block is deformed as shown. From our previous work in mechanics, we know that, provided the elastic limit of the solid material is not exceeded, the deformation is proportional to the applied shear stress, $\tau = F/A$, where A is the area of the surface in contact with the plate.

To repeat the experiment with a fluid between the plates, use a dye marker to outline a fluid element as shown by the solid lines (Fig. 1.1*b*). When the shear force, F, is applied to the upper plate, the deformation of the fluid element continues to increase as long as the force is applied. The fluid in direct contact with the solid boundary has the same velocity as the boundary itself; there is no slip at the boundary. This is an experimental fact based on numerous observations of fluid behavior.[1] The shape of the fluid element, at successive instants of time $t_2 > t_1 > t_0$, is shown (Fig. 1.1*b*) by the dashed lines, which represent the positions of the dye markers at successive times. Because the fluid motion continues under the application of a shear stress, we can also define a fluid as a substance that cannot sustain a shear stress when at rest.



(a) Solid (b) Fluid

Fig. 1.1 Behavior of a solid and a fluid, under the action of a constant shear force.

[1] The no-slip condition is demonstrated in the NCFMF video *Fundamentals of Boundary Layers*. A complete list of fluid mechanics video titles and sources is given in Appendix C.

1-3 SCOPE OF FLUID MECHANICS

Fluid mechanics deals with the behavior of fluids at rest and in motion. We might ask the question, "Why study fluid mechanics?"

Knowledge and understanding of the basic principles and concepts of fluid mechanics are essential to analyze any system in which a fluid is the working medium. We can give many examples. The design of virtually all means of transportation requires application of the principles of fluid mechanics. Included are subsonic and supersonic aircraft, surface ships, submarines, and automobiles. In recent years automobile manufacturers have given more consideration to aerodynamic design. This has been true for some time for the designers of both racing cars and boats. The design of propulsion systems for space flight as well as for toy rockets is based on the principles of fluid mechanics. The collapse of the Tacoma Narrows Bridge in 1940 is evidence of the possible consequences of neglecting the basic principles of fluid mechanics.[2] It is commonplace today to perform model studies to determine the aerodynamic forces on, and flow fields around, buildings and structures. These include studies of skyscrapers, baseball stadiums, smokestacks, and shopping plazas.

The design of all types of fluid machinery including pumps, fans, blowers, compressors, and turbines clearly requires knowledge of the basic principles of fluid mechanics. Lubrication is an application of considerable importance in fluid mechanics. Heating and ventilating systems for private homes and large office buildings and the design of pipeline systems are further examples of technical problem areas requiring knowledge of fluid mechanics. The circulatory system of the body is essentially a fluid system. It is not surprising that the design of blood substitutes, artificial hearts, heart-lung machines, breathing aids, and other such devices must rely on the basic principles of fluid mechanics.

Even some of our recreational endeavors are directly related to fluid mechanics. The slicing and hooking of golf balls can be explained by the principles of fluid mechanics (although they can be corrected only by a golf pro!).

This list of real-world applications of fluid mechanics could go on indefinitely. Our main point here is that fluid mechanics is not a subject studied for purely academic interest; rather, it is a subject with widespread importance both in our everyday experiences and in modern technology.

Clearly, we cannot hope to consider in detail even a small percentage of these and other specific problems of fluid mechanics. Instead, the purpose of this text is to present the basic laws and associated physical concepts that provide the basis or starting point in the analysis of any problem in fluid mechanics.

1-4 BASIC EQUATIONS

Analysis of any problem in fluid mechanics necessarily includes statement of the basic laws governing the fluid motion. The basic laws, which are applicable to any fluid, are:

[2] For dramatic evidence of aerodynamic forces in action, see the short video *Collapse of the Tacoma Narrows Bridge*.

1. The conservation of mass.
2. Newton's second law of motion.
3. The principle of angular momentum.
4. The first law of thermodynamics.
5. The second law of thermodynamics.

Not all basic laws are always required to solve any one problem. On the other hand, in many problems it is necessary to bring into the analysis additional relations that describe the behavior of physical properties of fluids under given conditions.

For example, you probably recall studying properties of gases in basic physics or thermodynamics. The *ideal gas* equation of state

$$p = \rho RT \tag{1.1}$$

is a model that relates density to pressure and temperature for many gases under normal conditions. In Eq. 1.1, R is the gas constant. Values of R are given in Appendix A for several common gases; p and T in Eq. 1.1 are the absolute pressure and absolute temperature, respectively; ρ is density (mass per unit volume). Example Problem 1.1 illustrates use of the ideal gas equation of state.

It is obvious that the basic laws with which we shall deal are the same as those used in mechanics and thermodynamics. Our task will be to formulate these laws in suitable forms to solve fluid flow problems and to apply them to a wide variety of situations.

We must emphasize that there are, as we shall see, many apparently simple problems in fluid mechanics that cannot be solved analytically. In such cases we must resort to more complicated numerical solutions and/or results of experimental tests.

1-5 METHODS OF ANALYSIS

The first step in solving a problem is to define the system that you are attempting to analyze. In basic mechanics, we made extensive use of the *free-body diagram*. We will use a *system* or a *control volume*, depending on the problem being studied. These concepts are identical to the ones you used in thermodynamics (except you may have called them *closed system* and *open system*, respectively). We can use either one to get mathematical expressions for each of the basic laws. In thermodynamics they were mostly used to obtain expressions for conservation of mass and the first and second laws of thermodynamics; in our study of fluid mechanics, we will be most interested in conservation of mass and Newton's second law of motion. In thermodynamics our focus was energy; in fluid mechanics it will mainly be forces and motion. We must always be aware of whether we are using a system or a control volume approach because each leads to different mathematical expressions of these laws. At this point we review the definitions of systems and control volumes.

System and Control Volume

A *system* is defined as a fixed, identifiable quantity of mass; the system boundaries separate the system from the surroundings. The boundaries of the system may be fixed or movable; however, no mass crosses the system boundaries.

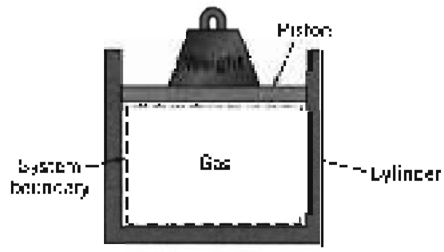


Fig. 1.2 Piston-cylinder assembly

In the familiar piston-cylinder assembly from thermodynamics, Fig. 1.2, the gas in the cylinder is the system. If the gas is heated, the piston will lift the weight; the boundary of the system thus moves. Heat and work may cross the boundaries of the system, but the quantity of matter within the system boundaries remains fixed. No mass crosses the system boundaries.

EXAMPLE 1.1 First Law Application to Closed System

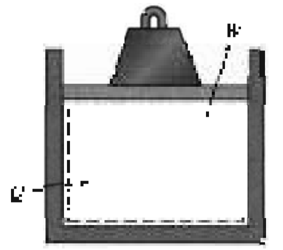
A piston-cylinder device contains 0.95 kg of oxygen initially at a temperature of 27°C and a pressure due to the weight of 150 kPa (abs). Heat is added to the gas until it reaches a temperature of 627°C. Determine the amount of heat added during the process.

EXAMPLE PROBLEM 1.1

GIVEN: Piston-cylinder containing O_2, $m = 0.95$ kg.

$$T_1 = 27°\text{C} \qquad T_2 = 627°\text{C}$$

FIND: $Q_{1\to 2}$



SOLUTION:

$p = \text{constant} = 150$ kPa (abs)

We are dealing with a system, $m = 0.95$ kg.

Governing equation: First law for the system, $Q_{12} - W_{12} = E_2 - E_1$

Assumptions: (1) $E = U$, since the system is stationary
(2) Ideal gas with constant specific heats

Under the above assumptions,

$$E_2 - E_1 = U_2 - U_1 = m(u_2 - u_1) = mc_v(T_2 - T_1)$$

The work done during the process is moving boundary work

$$W_{12} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

For an ideal gas, $pV = mRT$. Hence $W_{12} = mR(T_2 - T_1)$. Then from the first law equation,

$$Q_{12} = E_2 - E_1 + W_{12} = mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

$$Q_{12} = m(T_2 - T_1)(c_v + R)$$

$$Q_{12} = mc_p(T_2 - T_1) \qquad \{R = c_p - c_v\}$$

From the Appendix, Table A.6, for O_2, $c_p = 909.4$ J/(kg · K). Solving for Q_{12}, we obtain

$$Q_{12} = 0.95\ \text{kg} \times 909\ \frac{\text{J}}{\text{kg}\cdot\text{K}} \times 600\ \text{K} = 518\ \text{kJ} \longleftarrow Q_{12}$$

> This problem:
> ✓ Was solved using the nine logical steps discussed earlier.
> ✓ Reviewed use of the ideal gas equation and the first law of thermodynamics for a system.

In mechanics courses you used the free-body diagram (system approach) extensively. This was logical because you were dealing with an easily identifiable rigid body. However, in fluid mechanics we normally are concerned with the flow of fluids through devices such as compressors, turbines, pipelines, nozzles, and so on. In these cases it is difficult to focus attention on a fixed identifiable quantity of mass. It is much more convenient, for analysis, to focus attention on a volume in space through which the fluid flows. Consequently, we use the control volume approach.

A *control volume* is an arbitrary volume in space through which fluid flows. The geometric boundary of the control volume is called the control surface. The control surface may be real or imaginary; it may be at rest or in motion. Figure 1.3 shows flow through a pipe junction, with a control surface drawn on it. Note that some regions of the surface correspond to physical boundaries (the walls of the pipe) and others (at locations ①, ②, and ③) are parts of the surface that are imaginary (inlets or outlets). For the control volume defined by this surface, we could write equations for the basic laws and obtain results such as the flow rate at outlet ③ given the flow rates at inlet ① and outlet ② (similar to a problem we will analyze in Example Problem 4.1 in Chapter 4), the force required to hold the junction in place, and so on.

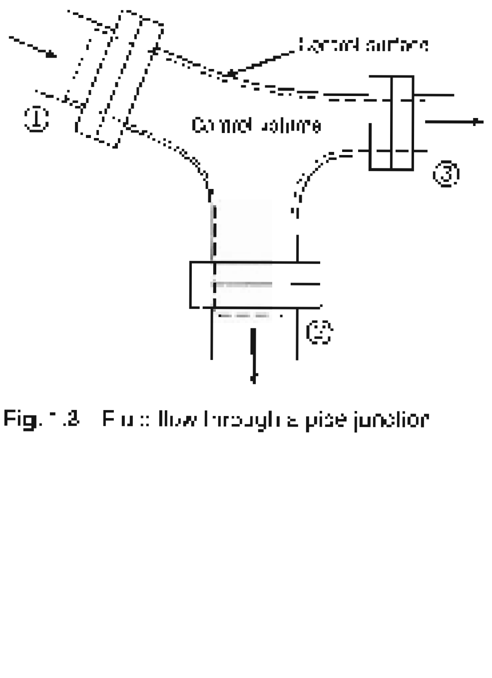


Fig. 1.3 Fluid flow through a pipe junction.

It is always important to take care in selecting a control volume, as the choice has a big effect on the mathematical form of the basic laws.

Differential versus Integral Approach

The basic laws that we apply in our study of fluid mechanics can be formulated in terms of *infinitesimal* or *finite* systems and control volumes. As you might suspect, the equations will look different in the two cases. Both approaches are important in the study of fluid mechanics and both will be developed in the course of our work.

In the first case the resulting equations are differential equations. Solution of the differential equations of motion provides a means of determining the detailed behavior of the flow. An example might be the pressure distribution on a wing surface.

Frequently the information sought does not require a detailed knowledge of the flow. We often are interested in the gross behavior of a device; in such cases it is more appropriate to use integral formulations of the basic laws. An example might be the overall lift a wing produces. Integral formulations, using finite systems or control volumes, usually are easier to treat analytically. The basic laws of mechanics and thermodynamics, formulated in terms of finite systems, are the basis for deriving the control volume equations in Chapter 4.

Methods of Description

Mechanics deals almost exclusively with systems; you have made extensive use of the basic equations applied to a fixed, identifiable quantity of mass. On the other hand, attempting to analyze thermodynamic devices, you often found it necessary to use a control volume (open system) analysis. Clearly, the type of analysis depends on the problem.

Where it is easy to keep track of identifiable elements of mass (e.g., in particle mechanics), we use a method of description that follows the particle. This sometimes is referred to as the *Lagrangian* method of description.

Consider, for example, the application of Newton's second law to a particle of fixed mass. Mathematically, we can write Newton's second law for a system of mass m as

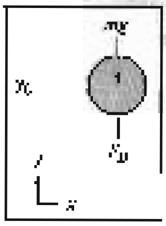
$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = m\frac{d^2\vec{r}}{dt^2} \tag{1.2}$$

In Eq. 1.2, $\sum \vec{F}$ is the sum of all external forces acting on the system, $\vec{a}$ is the acceleration of the center of mass of the system, $\vec{V}$ is the velocity of the center of mass of the system, and $\vec{r}$ is the position vector of the center of mass of the system relative to a fixed coordinate system.

EXAMPLE 1.2 Free-Fall of Ball in Air

The air resistance (drag force) on a 200 g ball in free flight is given by $F_D = 2 \times 10^{-4} V^2$, where F_D is in newtons and V is in meters per second. If the ball is dropped from rest 500 m above the ground, determine the speed at which it hits the ground. What percentage of the terminal speed is the result? (The *terminal speed* is the steady speed a falling body eventually attains.)

EXAMPLE PROBLEM 1.2



GIVEN: Ball, $m = 0.2$ kg, released from rest at $y_0 = 500$ m.
Air resistance, $F_D = kV^2$, where $k = 2 \times 10^{-4}\ \mathrm{N \cdot s^2/m^2}$

Units: F_D(N), V(m/s)

FIND: (a) Speed at which the ball hits the ground.
(b) Ratio of speed to terminal speed.

SOLUTION:

Governing equation: $\sum \vec{F} = m\vec{a}$

Assumption: (1) Neglect buoyancy force.

The motion of the ball is governed by the equation

$$\sum F_y = ma_y = m\frac{dV}{dt}$$

Since $V = V(y)$, we write $\sum F_y = m\dfrac{dV}{dy}\dfrac{dy}{dt} = mV\dfrac{dV}{dy}$. Then,

$$\sum F_y = F_D - mg = kV^2 - mg = mV\frac{dV}{dy}$$

Separating variables and integrating,

$$\int_{y_0}^{y} dy = \int_0^V \frac{mV\,dV}{kV^2 - mg}$$

$$y - y_0 = \left[\frac{m}{2k}\ln(kV^2 - mg)\right]_0^V = \frac{m}{2k}\ln\frac{kV^2 - mg}{-mg}$$

Taking antilogarithms, we obtain

$$kV^2 - mg = -mg\,e^{\left[\frac{2k}{m}(y-y_0)\right]}$$

Solving for V gives

$$V = \left\{\frac{mg}{k}\left(1 - e^{\left[\frac{2k}{m}(y-y_0)\right]}\right)\right\}^{1/2}$$

Substituting numerical values with $y = 0$ yields

$$V = \left\{0.2\,\mathrm{kg} \times 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2} \times \frac{\mathrm{m}^2}{2 \times 10^{-4}\,\mathrm{N \cdot s^2}} \times \frac{\mathrm{N \cdot s^2}}{\mathrm{kg \cdot m}}\left(1 - e^{\left[\frac{2 \times 2 \times 10^{-4}}{0.2}(-500)\right]}\right)\right\}^{1/2}$$

$V = 78.7$ m/s $\longleftarrow$ V