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# An Introduction to Categorical Data Analysis

Second Edition

ALAN AGRESTI

Department of Statistics  
University of Florida  
Gainesville, Florida



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# Contents

<b>Preface to the Second Edition</b>	<b>xv</b>
<b>1. Introduction</b>	<b>1</b>
1.1 Categorical Response Data, 1	
1.1.1 Response/Explanatory Variable Distinction, 2	
1.1.2 Nominal/Ordinal Scale Distinction, 2	
1.1.3 Organization of this Book, 3	
1.2 Probability Distributions for Categorical Data, 3	
1.2.1 Binomial Distribution, 4	
1.2.2 Multinomial Distribution, 5	
1.3 Statistical Inference for a Proportion, 6	
1.3.1 Likelihood Function and Maximum Likelihood Estimation, 6	
1.3.2 Significance Test About a Binomial Proportion, 8	
1.3.3 Example: Survey Results on Legalizing Abortion, 8	
1.3.4 Confidence Intervals for a Binomial Proportion, 9	
1.4 More on Statistical Inference for Discrete Data, 11	
1.4.1 Wald, Likelihood-Ratio, and Score Inference, 11	
1.4.2 Wald, Score, and Likelihood-Ratio Inference for Binomial Parameter, 12	
1.4.3 Small-Sample Binomial Inference, 13	
1.4.4 Small-Sample Discrete Inference is Conservative, 14	
1.4.5 Inference Based on the Mid $P$ -value, 15	
1.4.6 Summary, 16	
Problems, 16	
<b>2. Contingency Tables</b>	<b>21</b>
2.1 Probability Structure for Contingency Tables, 21	
2.1.1 Joint, Marginal, and Conditional Probabilities, 22	
2.1.2 Example: Belief in Afterlife, 22	

v



- 2.1.3 Sensitivity and Specificity in Diagnostic Tests, 23
- 2.1.4 Independence, 24
- 2.1.5 Binomial and Multinomial Sampling, 25
- 2.2 Comparing Proportions in Two-by-Two Tables, 25
  - 2.2.1 Difference of Proportions, 26
  - 2.2.2 Example: Aspirin and Heart Attacks, 26
  - 2.2.3 Relative Risk, 27
- 2.3 The Odds Ratio, 28
  - 2.3.1 Properties of the Odds Ratio, 29
  - 2.3.2 Example: Odds Ratio for Aspirin Use and Heart Attacks, 30
  - 2.3.3 Inference for Odds Ratios and Log Odds Ratios, 30
  - 2.3.4 Relationship Between Odds Ratio and Relative Risk, 32
  - 2.3.5 The Odds Ratio Applies in Case–Control Studies, 32
  - 2.3.6 Types of Observational Studies, 34
- 2.4 Chi-Squared Tests of Independence, 34
  - 2.4.1 Pearson Statistic and the Chi-Squared Distribution, 35
  - 2.4.2 Likelihood-Ratio Statistic, 36
  - 2.4.3 Tests of Independence, 36
  - 2.4.4 Example: Gender Gap in Political Affiliation, 37
  - 2.4.5 Residuals for Cells in a Contingency Table, 38
  - 2.4.6 Partitioning Chi-Squared, 39
  - 2.4.7 Comments About Chi-Squared Tests, 40
- 2.5 Testing Independence for Ordinal Data, 41
  - 2.5.1 Linear Trend Alternative to Independence, 41
  - 2.5.2 Example: Alcohol Use and Infant Malformation, 42
  - 2.5.3 Extra Power with Ordinal Tests, 43
  - 2.5.4 Choice of Scores, 43
  - 2.5.5 Trend Tests for  $I \times 2$  and  $2 \times J$  Tables, 44
  - 2.5.6 Nominal–Ordinal Tables, 45
- 2.6 Exact Inference for Small Samples, 45
  - 2.6.1 Fisher’s Exact Test for  $2 \times 2$  Tables, 45
  - 2.6.2 Example: Fisher’s Tea Taster, 46
  - 2.6.3  $P$ -values and Conservatism for Actual  $P$ (Type I Error), 47
  - 2.6.4 Small-Sample Confidence Interval for Odds Ratio, 48
- 2.7 Association in Three-Way Tables, 49
  - 2.7.1 Partial Tables, 49
  - 2.7.2 Conditional Versus Marginal Associations: Death Penalty Example, 49
  - 2.7.3 Simpson’s Paradox, 51
  - 2.7.4 Conditional and Marginal Odds Ratios, 52
  - 2.7.5 Conditional Independence Versus Marginal Independence, 53
  - 2.7.6 Homogeneous Association, 54
- Problems, 55

**3. Generalized Linear Models****65**

- 3.1 Components of a Generalized Linear Model, 66
    - 3.1.1 Random Component, 66
    - 3.1.2 Systematic Component, 66
    - 3.1.3 Link Function, 66
    - 3.1.4 Normal GLM, 67
  - 3.2 Generalized Linear Models for Binary Data, 68
    - 3.2.1 Linear Probability Model, 68
    - 3.2.2 Example: Snoring and Heart Disease, 69
    - 3.2.3 Logistic Regression Model, 70
    - 3.2.4 Probit Regression Model, 72
    - 3.2.5 Binary Regression and Cumulative Distribution Functions, 72
  - 3.3 Generalized Linear Models for Count Data, 74
    - 3.3.1 Poisson Regression, 75
    - 3.3.2 Example: Female Horseshoe Crabs and their Satellites, 75
    - 3.3.3 Overdispersion: Greater Variability than Expected, 80
    - 3.3.4 Negative Binomial Regression, 81
    - 3.3.5 Count Regression for Rate Data, 82
    - 3.3.6 Example: British Train Accidents over Time, 83
  - 3.4 Statistical Inference and Model Checking, 84
    - 3.4.1 Inference about Model Parameters, 84
    - 3.4.2 Example: Snoring and Heart Disease Revisited, 85
    - 3.4.3 The Deviance, 85
    - 3.4.4 Model Comparison Using the Deviance, 86
    - 3.4.5 Residuals Comparing Observations to the Model Fit, 87
  - 3.5 Fitting Generalized Linear Models, 88
    - 3.5.1 The Newton–Raphson Algorithm Fits GLMs, 88
    - 3.5.2 Wald, Likelihood-Ratio, and Score Inference Use the Likelihood Function, 89
    - 3.5.3 Advantages of GLMs, 90
- Problems, 90

**4. Logistic Regression****99**

- 4.1 Interpreting the Logistic Regression Model, 99
  - 4.1.1 Linear Approximation Interpretations, 100
  - 4.1.2 Horseshoe Crabs: Viewing and Smoothing a Binary Outcome, 101
  - 4.1.3 Horseshoe Crabs: Interpreting the Logistic Regression Fit, 101
  - 4.1.4 Odds Ratio Interpretation, 104

- 4.1.5 Logistic Regression with Retrospective Studies, 105
  - 4.1.6 Normally Distributed  $X$  Implies Logistic Regression for  $Y$ , 105
  - 4.2 Inference for Logistic Regression, 106
    - 4.2.1 Binary Data can be Grouped or Ungrouped, 106
    - 4.2.2 Confidence Intervals for Effects, 106
    - 4.2.3 Significance Testing, 107
    - 4.2.4 Confidence Intervals for Probabilities, 108
    - 4.2.5 Why Use a Model to Estimate Probabilities?, 108
    - 4.2.6 Confidence Intervals for Probabilities: Details, 108
    - 4.2.7 Standard Errors of Model Parameter Estimates, 109
  - 4.3 Logistic Regression with Categorical Predictors, 110
    - 4.3.1 Indicator Variables Represent Categories of Predictors, 110
    - 4.3.2 Example: AZT Use and AIDS, 111
    - 4.3.3 ANOVA-Type Model Representation of Factors, 113
    - 4.3.4 The Cochran–Mantel–Haenszel Test for  $2 \times 2 \times K$  Contingency Tables, 114
    - 4.3.5 Testing the Homogeneity of Odds Ratios, 115
  - 4.4 Multiple Logistic Regression, 115
    - 4.4.1 Example: Horseshoe Crabs with Color and Width Predictors, 116
    - 4.4.2 Model Comparison to Check Whether a Term is Needed, 118
    - 4.4.3 Quantitative Treatment of Ordinal Predictor, 118
    - 4.4.4 Allowing Interaction, 119
  - 4.5 Summarizing Effects in Logistic Regression, 120
    - 4.5.1 Probability-Based Interpretations, 120
    - 4.5.2 Standardized Interpretations, 121
- Problems, 121

## 5. Building and Applying Logistic Regression Models 137

- 5.1 Strategies in Model Selection, 137
  - 5.1.1 How Many Predictors Can You Use?, 138
  - 5.1.2 Example: Horseshoe Crabs Revisited, 138
  - 5.1.3 Stepwise Variable Selection Algorithms, 139
  - 5.1.4 Example: Backward Elimination for Horseshoe Crabs, 140
  - 5.1.5 AIC, Model Selection, and the “Correct” Model, 141
  - 5.1.6 Summarizing Predictive Power: Classification Tables, 142
  - 5.1.7 Summarizing Predictive Power: ROC Curves, 143
  - 5.1.8 Summarizing Predictive Power: A Correlation, 144
- 5.2 Model Checking, 144
  - 5.2.1 Likelihood-Ratio Model Comparison Tests, 144
  - 5.2.2 Goodness of Fit and the Deviance, 145

- 5.2.3 Checking Fit: Grouped Data, Ungrouped Data, and Continuous Predictors, 146
  - 5.2.4 Residuals for Logit Models, 147
  - 5.2.5 Example: Graduate Admissions at University of Florida, 149
  - 5.2.6 Influence Diagnostics for Logistic Regression, 150
  - 5.2.7 Example: Heart Disease and Blood Pressure, 151
  - 5.3 Effects of Sparse Data, 152
    - 5.3.1 Infinite Effect Estimate: Quantitative Predictor, 152
    - 5.3.2 Infinite Effect Estimate: Categorical Predictors, 153
    - 5.3.3 Example: Clinical Trial with Sparse Data, 154
    - 5.3.4 Effect of Small Samples on  $X^2$  and  $G^2$  Tests, 156
  - 5.4 Conditional Logistic Regression and Exact Inference, 157
    - 5.4.1 Conditional Maximum Likelihood Inference, 157
    - 5.4.2 Small-Sample Tests for Contingency Tables, 158
    - 5.4.3 Example: Promotion Discrimination, 159
    - 5.4.4 Small-Sample Confidence Intervals for Logistic Parameters and Odds Ratios, 159
    - 5.4.5 Limitations of Small-Sample Exact Methods, 160
  - 5.5 Sample Size and Power for Logistic Regression, 160
    - 5.5.1 Sample Size for Comparing Two Proportions, 161
    - 5.5.2 Sample Size in Logistic Regression, 161
    - 5.5.3 Sample Size in Multiple Logistic Regression, 162
- Problems, 163

## 6. Multicategory Logit Models

173

- 6.1 Logit Models for Nominal Responses, 173
  - 6.1.1 Baseline-Category Logits, 173
  - 6.1.2 Example: Alligator Food Choice, 174
  - 6.1.3 Estimating Response Probabilities, 176
  - 6.1.4 Example: Belief in Afterlife, 178
  - 6.1.5 Discrete Choice Models, 179
- 6.2 Cumulative Logit Models for Ordinal Responses, 180
  - 6.2.1 Cumulative Logit Models with Proportional Odds Property, 180
  - 6.2.2 Example: Political Ideology and Party Affiliation, 182
  - 6.2.3 Inference about Model Parameters, 184
  - 6.2.4 Checking Model Fit, 184
  - 6.2.5 Example: Modeling Mental Health, 185
  - 6.2.6 Interpretations Comparing Cumulative Probabilities, 187
  - 6.2.7 Latent Variable Motivation, 187
  - 6.2.8 Invariance to Choice of Response Categories, 189
- 6.3 Paired-Category Ordinal Logits, 189

- 6.3.1 Adjacent-Categories Logits, 190
- 6.3.2 Example: Political Ideology Revisited, 190
- 6.3.3 Continuation-Ratio Logits, 191
- 6.3.4 Example: A Developmental Toxicity Study, 191
- 6.3.5 Overdispersion in Clustered Data, 192
- 6.4 Tests of Conditional Independence, 193
  - 6.4.1 Example: Job Satisfaction and Income, 193
  - 6.4.2 Generalized Cochran–Mantel–Haenszel Tests, 194
  - 6.4.3 Detecting Nominal–Ordinal Conditional Association, 195
  - 6.4.4 Detecting Nominal–Nominal Conditional Association, 196
- Problems, 196

## **7. Loglinear Models for Contingency Tables 204**

- 7.1 Loglinear Models for Two-Way and Three-Way Tables, 204
  - 7.1.1 Loglinear Model of Independence for Two-Way Table, 205
  - 7.1.2 Interpretation of Parameters in Independence Model, 205
  - 7.1.3 Saturated Model for Two-Way Tables, 206
  - 7.1.4 Loglinear Models for Three-Way Tables, 208
  - 7.1.5 Two-Factor Parameters Describe Conditional Associations, 209
  - 7.1.6 Example: Alcohol, Cigarette, and Marijuana Use, 209
- 7.2 Inference for Loglinear Models, 212
  - 7.2.1 Chi-Squared Goodness-of-Fit Tests, 212
  - 7.2.2 Loglinear Cell Residuals, 213
  - 7.2.3 Tests about Conditional Associations, 214
  - 7.2.4 Confidence Intervals for Conditional Odds Ratios, 214
  - 7.2.5 Loglinear Models for Higher Dimensions, 215
  - 7.2.6 Example: Automobile Accidents and Seat Belts, 215
  - 7.2.7 Three-Factor Interaction, 218
  - 7.2.8 Large Samples and Statistical vs Practical Significance, 218
- 7.3 The Loglinear–Logistic Connection, 219
  - 7.3.1 Using Logistic Models to Interpret Loglinear Models, 219
  - 7.3.2 Example: Auto Accident Data Revisited, 220
  - 7.3.3 Correspondence Between Loglinear and Logistic Models, 221
  - 7.3.4 Strategies in Model Selection, 221
- 7.4 Independence Graphs and Collapsibility, 223
  - 7.4.1 Independence Graphs, 223
  - 7.4.2 Collapsibility Conditions for Three-Way Tables, 224
  - 7.4.3 Collapsibility and Logistic Models, 225
  - 7.4.4 Collapsibility and Independence Graphs for Multiway Tables, 225
  - 7.4.5 Example: Model Building for Student Drug Use, 226
  - 7.4.6 Graphical Models, 228

- 7.5 Modeling Ordinal Associations, 228
  - 7.5.1 Linear-by-Linear Association Model, 229
  - 7.5.2 Example: Sex Opinions, 230
  - 7.5.3 Ordinal Tests of Independence, 232
- Problems, 232

## **8. Models for Matched Pairs 244**

- 8.1 Comparing Dependent Proportions, 245
  - 8.1.1 McNemar Test Comparing Marginal Proportions, 245
  - 8.1.2 Estimating Differences of Proportions, 246
- 8.2 Logistic Regression for Matched Pairs, 247
  - 8.2.1 Marginal Models for Marginal Proportions, 247
  - 8.2.2 Subject-Specific and Population-Averaged Tables, 248
  - 8.2.3 Conditional Logistic Regression for Matched-Pairs, 249
  - 8.2.4 Logistic Regression for Matched Case–Control Studies, 250
  - 8.2.5 Connection between McNemar and Cochran–Mantel–Haenszel Tests, 252
- 8.3 Comparing Margins of Square Contingency Tables, 252
  - 8.3.1 Marginal Homogeneity and Nominal Classifications, 253
  - 8.3.2 Example: Coffee Brand Market Share, 253
  - 8.3.3 Marginal Homogeneity and Ordered Categories, 254
  - 8.3.4 Example: Recycle or Drive Less to Help Environment?, 255
- 8.4 Symmetry and Quasi-Symmetry Models for Square Tables, 256
  - 8.4.1 Symmetry as a Logistic Model, 257
  - 8.4.2 Quasi-Symmetry, 257
  - 8.4.3 Example: Coffee Brand Market Share Revisited, 257
  - 8.4.4 Testing Marginal Homogeneity Using Symmetry and Quasi-Symmetry, 258
  - 8.4.5 An Ordinal Quasi-Symmetry Model, 258
  - 8.4.6 Example: Recycle or Drive Less?, 259
  - 8.4.7 Testing Marginal Homogeneity Using Symmetry and Ordinal Quasi-Symmetry, 259
- 8.5 Analyzing Rater Agreement, 260
  - 8.5.1 Cell Residuals for Independence Model, 261
  - 8.5.2 Quasi-independence Model, 261
  - 8.5.3 Odds Ratios Summarizing Agreement, 262
  - 8.5.4 Quasi-Symmetry and Agreement Modeling, 263
  - 8.5.5 Kappa Measure of Agreement, 264
- 8.6 Bradley–Terry Model for Paired Preferences, 264
  - 8.6.1 The Bradley–Terry Model, 265
  - 8.6.2 Example: Ranking Men Tennis Players, 265
- Problems, 266

<b>9. Modeling Correlated, Clustered Responses</b>	<b>276</b>
9.1 Marginal Models Versus Conditional Models, 277	
9.1.1 Marginal Models for a Clustered Binary Response, 277	
9.1.2 Example: Longitudinal Study of Treatments for Depression, 277	
9.1.3 Conditional Models for a Repeated Response, 279	
9.2 Marginal Modeling: The GEE Approach, 279	
9.2.1 Quasi-Likelihood Methods, 280	
9.2.2 Generalized Estimating Equation Methodology: Basic Ideas, 280	
9.2.3 GEE for Binary Data: Depression Study, 281	
9.2.4 Example: Teratology Overdispersion, 283	
9.2.5 Limitations of GEE Compared with ML, 284	
9.3 Extending GEE: Multinomial Responses, 285	
9.3.1 Marginal Modeling of a Clustered Multinomial Response, 285	
9.3.2 Example: Insomnia Study, 285	
9.3.3 Another Way of Modeling Association with GEE, 287	
9.3.4 Dealing with Missing Data, 287	
9.4 Transitional Modeling, Given the Past, 288	
9.4.1 Transitional Models with Explanatory Variables, 288	
9.4.2 Example: Respiratory Illness and Maternal Smoking, 288	
9.4.3 Comparisons that Control for Initial Response, 289	
9.4.4 Transitional Models Relate to Loglinear Models, 290	
Problems, 290	
<b>10. Random Effects: Generalized Linear Mixed Models</b>	<b>297</b>
10.1 Random Effects Modeling of Clustered Categorical Data, 297	
10.1.1 The Generalized Linear Mixed Model, 298	
10.1.2 A Logistic GLMM for Binary Matched Pairs, 299	
10.1.3 Example: Sacrifices for the Environment Revisited, 300	
10.1.4 Differing Effects in Conditional Models and Marginal Models, 300	
10.2 Examples of Random Effects Models for Binary Data, 302	
10.2.1 Small-Area Estimation of Binomial Probabilities, 302	
10.2.2 Example: Estimating Basketball Free Throw Success, 303	
10.2.3 Example: Teratology Overdispersion Revisited, 304	
10.2.4 Example: Repeated Responses on Similar Survey Items, 305	
10.2.5 Item Response Models: The Rasch Model, 307	
10.2.6 Example: Depression Study Revisited, 307	
10.2.7 Choosing Marginal or Conditional Models, 308	
10.2.8 Conditional Models: Random Effects Versus Conditional ML, 309	

10.3	Extensions to Multinomial Responses or Multiple Random Effect Terms, 310	
10.3.1	Example: Insomnia Study Revisited, 310	
10.3.2	Bivariate Random Effects and Association Heterogeneity, 311	
10.4	Multilevel (Hierarchical) Models, 313	
10.4.1	Example: Two-Level Model for Student Advancement, 314	
10.4.2	Example: Grade Retention, 315	
10.5	Model Fitting and Inference for GLMMS, 316	
10.5.1	Fitting GLMMs, 316	
10.5.2	Inference for Model Parameters and Prediction, 317	
	Problems, 318	
<b>11.</b>	<b>A Historical Tour of Categorical Data Analysis</b>	<b>325</b>
11.1	The Pearson–Yule Association Controversy, 325	
11.2	R. A. Fisher’s Contributions, 326	
11.3	Logistic Regression, 328	
11.4	Multiway Contingency Tables and Loglinear Models, 329	
11.5	Final Comments, 331	
	<b>Appendix A: Software for Categorical Data Analysis</b>	<b>332</b>
	<b>Appendix B: Chi-Squared Distribution Values</b>	<b>343</b>
	<b>Bibliography</b>	<b>344</b>
	<b>Index of Examples</b>	<b>346</b>
	<b>Subject Index</b>	<b>350</b>
	<b>Brief Solutions to Some Odd-Numbered Problems</b>	<b>357</b>





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## Preface to the Second Edition

In recent years, the use of specialized statistical methods for categorical data has increased dramatically, particularly for applications in the biomedical and social sciences. Partly this reflects the development during the past few decades of sophisticated methods for analyzing categorical data. It also reflects the increasing methodological sophistication of scientists and applied statisticians, most of whom now realize that it is unnecessary and often inappropriate to use methods for continuous data with categorical responses.

This book presents the most important methods for analyzing categorical data. It summarizes methods that have long played a prominent role, such as chi-squared tests. It gives special emphasis, however, to modeling techniques, in particular to logistic regression.

The presentation in this book has a low technical level and does not require familiarity with advanced mathematics such as calculus or matrix algebra. Readers should possess a background that includes material from a two-semester statistical methods sequence for undergraduate or graduate nonstatistics majors. This background should include estimation and significance testing and exposure to regression modeling.

This book is designed for students taking an introductory course in categorical data analysis, but I also have written it for applied statisticians and practicing scientists involved in data analyses. I hope that the book will be helpful to analysts dealing with categorical response data in the social, behavioral, and biomedical sciences, as well as in public health, marketing, education, biological and agricultural sciences, and industrial quality control.

The basics of categorical data analysis are covered in Chapters 1–8. Chapter 2 surveys standard descriptive and inferential methods for contingency tables, such as odds ratios, tests of independence, and conditional vs marginal associations. I feel that an understanding of methods is enhanced, however, by viewing them in the context of statistical models. Thus, the rest of the text focuses on the modeling of categorical responses. Chapter 3 introduces generalized linear models for binary data and count data. Chapters 4 and 5 discuss the most important such model for binomial (binary) data, logistic regression. Chapter 6 introduces logistic regression models

for multinomial responses, both nominal and ordinal. Chapter 7 discusses loglinear models for Poisson (count) data. Chapter 8 presents methods for matched-pairs data.

I believe that logistic regression is more important than loglinear models, since most applications with categorical responses have a single binomial or multinomial response variable. Thus, I have given main attention to this model in these chapters and in later chapters that discuss extensions of this model. Compared with the first edition, this edition places greater emphasis on logistic regression and less emphasis on loglinear models.

I prefer to teach categorical data methods by unifying their models with ordinary regression and ANOVA models. Chapter 3 does this under the umbrella of generalized linear models. Some instructors might prefer to cover this chapter rather lightly, using it primarily to introduce logistic regression models for binomial data (Sections 3.1 and 3.2).

The main change from the first edition is the addition of two chapters dealing with the analysis of clustered correlated categorical data, such as occur in longitudinal studies with repeated measurement of subjects. Chapters 9 and 10 extend the matched-pairs methods of Chapter 8 to apply to clustered data. Chapter 9 does this with marginal models, emphasizing the generalized estimating equations (GEE) approach, whereas Chapter 10 uses random effects to model more fully the dependence. The text concludes with a chapter providing a historical perspective of the development of the methods (Chapter 11) and an appendix showing the use of SAS for conducting nearly all methods presented in this book.

The material in Chapters 1–8 forms the heart of an introductory course in categorical data analysis. Sections that can be skipped if desired, to provide more time for other topics, include Sections 2.5, 2.6, 3.3 and 3.5, 5.3–5.5, 6.3, 6.4, 7.4, 7.5, and 8.3–8.6. Instructors can choose sections from Chapters 9–11 to supplement the basic topics in Chapters 1–8. Within sections, subsections labelled with an asterisk are less important and can be skipped for those wanting a quick exposure to the main points.

This book is of a lower technical level than my book *Categorical Data Analysis* (2nd edition, Wiley, 2002). I hope that it will appeal to readers who prefer a more applied focus than that book provides. For instance, this book does not attempt to derive likelihood equations, prove asymptotic distributions, discuss current research work, or present a complete bibliography.

Most methods presented in this text require extensive computations. For the most part, I have avoided details about complex calculations, feeling that computing software should relieve this drudgery. Software for categorical data analyses is widely available in most large commercial packages. I recommend that readers of this text use software wherever possible in answering homework problems and checking text examples. The Appendix discusses the use of SAS (particularly PROC GENMOD) for nearly all methods discussed in the text. The tables in the Appendix and many of the data sets analyzed in the book are available at the web site <http://www.stat.ufl.edu/~aa/intro-cda/appendix.html>. The web site <http://www.stat.ufl.edu/~aa/cda/software.html> contains information about the use of other software, such as S-Plus and R, Stata, and SPSS, including a link to an excellent free manual prepared by Laura Thompson showing how to use R and S-Plus to

conduct nearly all the examples in this book and its higher-level companion. Also listed at the text website are known typos and errors in early printings of the text.

I owe very special thanks to Brian Marx for his many suggestions about the text over the past 10 years. He has been incredibly generous with his time in providing feedback based on using the book many times in courses. He and Bernhard Klingenberg also very kindly reviewed the draft for this edition and made many helpful suggestions. I also thank those individuals who commented on parts of the manuscript or who made suggestions about examples or material to cover. These include Anna Gottard for suggestions about Section 7.4, Judy Breiner, Brian Caffo, Allen Hammer, and Carla Rampichini. I also owe thanks to those who helped with the first edition, especially Patricia Altham, James Booth, Jane Brockmann, Brent Coull, Al DeMaris, Joan Hilton, Peter Imrey, Harry Khamis, Svend Kreiner, Stephen Stigler, and Larry Winner. Thanks finally to those who helped with material for my more advanced text (*Categorical Data Analysis*) that I extracted here, especially Bernhard Klingenberg, Yongyi Min, and Brian Caffo. Many thanks to Stephen Quigley at Wiley for his continuing interest, and to the Wiley staff for their usual high-quality support.

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*January 2007*



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## CHAPTER 1

# Introduction

From helping to assess the value of new medical treatments to evaluating the factors that affect our opinions on various controversial issues, scientists today are finding myriad uses for methods of analyzing categorical data. It's primarily for these scientists and their collaborating statisticians – as well as those training to perform these roles – that this book was written. The book provides an introduction to methods for analyzing categorical data. It emphasizes the ideas behind the methods and their interpretations, rather than the theory behind them.

This first chapter reviews the probability distributions most often used for categorical data, such as the *binomial distribution*. It also introduces *maximum likelihood*, the most popular method for estimating parameters. We use this estimate and a related *likelihood function* to conduct statistical inference about proportions. We begin by discussing the major types of categorical data and summarizing the book's outline.

### 1.1 CATEGORICAL RESPONSE DATA

Let us first define categorical data. A *categorical* variable has a measurement scale consisting of a set of categories. For example, political philosophy may be measured as “liberal,” “moderate,” or “conservative”; choice of accommodation might use categories “house,” “condominium,” “apartment”; a diagnostic test to detect e-mail spam might classify an incoming e-mail message as “spam” or “legitimate e-mail.”

Categorical scales are pervasive in the social sciences for measuring attitudes and opinions. Categorical scales also occur frequently in the health sciences, for measuring responses such as whether a patient survives an operation (yes, no), severity of an injury (none, mild, moderate, severe), and stage of a disease (initial, advanced).

Although categorical variables are common in the social and health sciences, they are by no means restricted to those areas. They frequently occur in the behavioral

sciences (e.g., categories “schizophrenia,” “depression,” “neurosis” for diagnosis of type of mental illness), public health (e.g., categories “yes” and “no” for whether awareness of AIDS has led to increased use of condoms), zoology (e.g., categories “fish,” “invertebrate,” “reptile” for alligators’ primary food choice), education (e.g., categories “correct” and “incorrect” for students’ responses to an exam question), and marketing (e.g., categories “Brand A,” “Brand B,” and “Brand C” for consumers’ preference among three leading brands of a product). They even occur in highly quantitative fields such as engineering sciences and industrial quality control, when items are classified according to whether or not they conform to certain standards.

### 1.1.1 Response/Explanatory Variable Distinction

Most statistical analyses distinguish between *response* variables and *explanatory* variables. For instance, regression models describe how the distribution of a continuous response variable, such as annual income, changes according to levels of explanatory variables, such as number of years of education and number of years of job experience. The response variable is sometimes called the *dependent variable* or *Y variable*, and the explanatory variable is sometimes called the *independent variable* or *X variable*.

The subject of this text is the analysis of categorical response variables. The categorical variables listed in the previous subsection are response variables. In some studies, they might also serve as explanatory variables. Statistical models for categorical response variables analyze how such responses are influenced by explanatory variables. For example, a model for political philosophy could use predictors such as annual income, attained education, religious affiliation, age, gender, and race. The explanatory variables can be categorical or continuous.

### 1.1.2 Nominal/Ordinal Scale Distinction

Categorical variables have two main types of measurement scales. Many categorical scales have a natural ordering. Examples are attitude toward legalization of abortion (disapprove in all cases, approve only in certain cases, approve in all cases), appraisal of a company’s inventory level (too low, about right, too high), response to a medical treatment (excellent, good, fair, poor), and frequency of feeling symptoms of anxiety (never, occasionally, often, always). Categorical variables having ordered scales are called *ordinal* variables.

Categorical variables having unordered scales are called *nominal* variables. Examples are religious affiliation (categories Catholic, Jewish, Protestant, Muslim, other), primary mode of transportation to work (automobile, bicycle, bus, subway, walk), favorite type of music (classical, country, folk, jazz, rock), and favorite place to shop (local mall, local downtown, Internet, other).

For nominal variables, the order of listing the categories is irrelevant. The statistical analysis should not depend on that ordering. Methods designed for nominal variables give the same results no matter how the categories are listed. Methods designed for

ordinal variables utilize the category ordering. Whether we list the categories from low to high or from high to low is irrelevant in terms of substantive conclusions, but results of ordinal analyses would change if the categories were reordered in any other way.

Methods designed for ordinal variables *cannot* be used with nominal variables, since nominal variables do not have ordered categories. Methods designed for nominal variables *can* be used with nominal or ordinal variables, since they only require a categorical scale. When used with ordinal variables, however, they do not use the information about that ordering. This can result in serious loss of power. It is usually best to apply methods appropriate for the actual scale.

Categorical variables are often referred to as *qualitative*, to distinguish them from numerical-valued or *quantitative* variables such as weight, age, income, and number of children in a family. However, we will see it is often advantageous to treat ordinal data in a quantitative manner, for instance by assigning ordered scores to the categories.

### 1.1.3 Organization of this Book

Chapters 1 and 2 describe some standard methods of categorical data analysis developed prior to about 1960. These include basic analyses of association between two categorical variables.

Chapters 3–7 introduce models for categorical responses. These models resemble regression models for continuous response variables. In fact, Chapter 3 shows they are special cases of a generalized class of linear models that also contains the usual normal-distribution-based regression models. The main emphasis in this book is on *logistic regression* models. Applying to response variables that have two outcome categories, they are the focus of Chapters 4 and 5. Chapter 6 presents extensions to multicategory responses, both nominal and ordinal. Chapter 7 introduces *loglinear* models, which analyze associations among multiple categorical response variables.

The methods in Chapters 1–7 assume that observations are independent. Chapters 8–10 discuss logistic regression models that apply when some observations are correlated, such as with repeated measurement of subjects in longitudinal studies. An important special case is matched pairs that result from observing a categorical response for the same subjects at two separate times. The book concludes (Chapter 11) with a historical overview of categorical data methods.

Most methods for categorical data analysis require extensive computations. The Appendix discusses the use of SAS statistical software. A companion website for the book, <http://www.stat.ufl.edu/~aa/intro-cda/software.html>, discusses other software.

## 1.2 PROBABILITY DISTRIBUTIONS FOR CATEGORICAL DATA

Inferential statistical analyses require assumptions about the probability distribution of the response variable. For regression and analysis of variance (ANOVA)



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